Section 2: Parametric differentiation and integration

Exercise level 2

1. The parametric equations of a curve are $x = t(t^2 + 1),\ y = t^2 - 1$.
   Find the equation of the tangents at the points at which the curve crosses the *x*-axis.

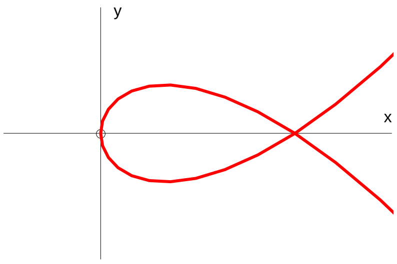
2. The tangent to the curve with parametric equations $x = t^2,\ y = 2t$ meets the *x* axis at A and the *y* axis at B. Find the coordinates of A and B in terms of *t*.

3. The parametric equations of a curve are $x = 4t,\ y = \frac{4}{t}$. The normal at $\mathrm{P}(8, 2)$ meets the curve again at Q. Find the coordinates of Q.

4. The parametric equations of an ellipse are $x = 2\cos\theta,\ y = 3\sin\theta$.
   (i) Find the points of intersections with the axes.
   (ii) Find the coordinates of the points for which the tangent to the curve cuts the *x*-axis at (4, 0).

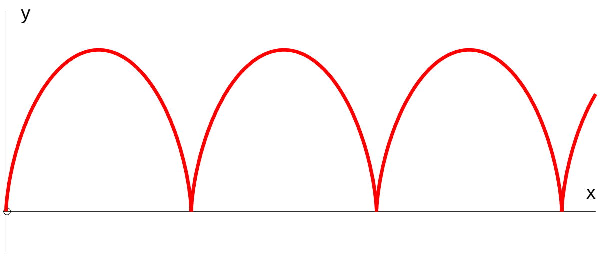
5. A curve has parametric equations $x = 2(\theta - \sin\theta),\ y = 2(1 - \cos\theta)$ for $0 \le \theta \le \pi$.
   (i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin\theta}{1 - \cos\theta}$
   (ii) The normal at the point P, at which $\theta = \frac{\pi}{2}$, meets the *x* axis at A.
   Find the exact coordinates of A.
   (iii) Write down the equation of the line through A which is parallel to the *y* axis. Find the point of intersection of this line with the tangent at P.

6. The curve shown below has parametric equations $x = 4t^2,\ y = 2t(1 - t^2)$.



   (i) Use the parametric equations to find the area within the loop of the curve.
   (ii) By eliminating *t*, find the Cartesian equation of the curve.
   (iii) Use the Cartesian equation to find the area within the loop of the curve. Check that this is the same as your answer to (i).

7. The diagram shows the curve given by the parametric equations
   $$x = \theta - \sin\theta, \quad y = 1 - \cos\theta.$$
   Find the area between the *x*-axis and one loop of the curve.



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