

Section 3: The product and quotient rules**Exercise level 3 (Extension)**

1. (i) Given that $f(x) = (x-a)^2 g(x)$, show that $f'(a) = 0$.

(ii) Given further that $g(a) > 0$, determine the nature of the stationary point at $x = a$.

2. (i) Use the quotient rule to find $\frac{dy}{dx}$ for the curve $y = \frac{x+1}{x-1}$.

(ii) By first writing $\frac{x+1}{x-1} = \frac{x-1+2}{x-1} = 1 + \frac{2}{x-1}$, obtain the result in (i) using the chain rule.

3. (i) Given $y = \frac{1}{g(x)}$, use the chain rule to show that $\frac{dy}{dx} = -\frac{g'(x)}{[g(x)]^2}$.

(ii) Now suppose $y = \frac{f(x)}{g(x)}$. By writing y as $f(x) \times \frac{1}{g(x)}$, use the product rule to find $\frac{dy}{dx}$. By expressing this as a single fraction, derive the quotient rule formula for differentiation.