Section 3: The product and quotient rules

Exercise level 2

1. (i) Using the product rule, differentiate $x(2x-1)^2$.
   (ii) Expand $x(2x-1)^2$. Then find the derivative of the resulting expression.
   (iii) Verify that your answers for parts (i) and (ii) are algebraically equivalent.

2. (i) Using the quotient rule, differentiate $\dfrac{5x^2}{\sqrt{x+3}}$.
   (ii) Rewrite $\dfrac{5x^2}{\sqrt{x+3}}$ as $5x^2(x+3)^{-\frac{1}{2}}$ and differentiate it using the product rule.
   (iii) Show your answers are algebraically equivalent.
   (iv) Explain why the function is not defined for $x \le -3$.

3. $y = x\sqrt{2x+5}$ for $x \ge -2.5$. Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ is of the form $\dfrac{ax+b}{\sqrt{2x+5}}$, where a and b are integers.

4. $y = \dfrac{x}{\sqrt{3x-2}}$ for $x > \frac{2}{3}$. Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ is of the form $\dfrac{ax+b}{(3x-2)^{\frac{3}{2}}}$, where a and b are rational numbers.

5. Find the exact coordinates of the turning point of the curve $y = x\sqrt{1-x}$.

6. Given that $y = \dfrac{x}{2x-1}$ for $x \ne \frac{1}{2}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{(2x-1)^2}$. Deduce that the gradient of the curve is always negative for $x \ne \frac{1}{2}$.

7. Find the equation of the tangent to $y = x^2\sqrt{2x-1}$, $x > \frac{1}{2}$, at the point where $x = 1$.

8. Find the coordinates of the turning points of the curve $y = \dfrac{x^2}{x-1}$.

9. Find the gradient of the curve $y = \dfrac{\sqrt{2x-1}}{x}$, $x > \frac{1}{2}$, at the point where $x = 2$.

10. (i) Given that $w = \dfrac{1+x}{1-x}$, find $\dfrac{\mathrm{d}w}{\mathrm{d}x}$.
    (ii) Using this result, and the chain rule, show that if $y = \left(\dfrac{1+x}{1-x}\right)^3$,
    then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6(1+x)^2}{(1-x)^4}$.