Section 2: The chain rule

Exercise level 2

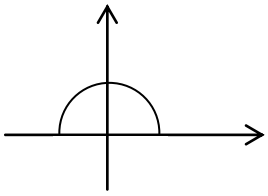
1. (i) Using the chain rule, differentiate $(2x - 1)^3$.
   (ii) Expand $(2x - 1)^3$ using the binomial theorem.
   Hence find the derivative of the resulting expression.
   (iii) Verify that your answers for parts (i) and (ii) are algebraically equivalent.

2. The equation of a curve is given by $y = (2x + 1)^4$.
   (i) Using the chain rule, find the gradient of the curve at the point (0, 1).
   (ii) Find the coordinates of the point where the gradient is zero.

3. Find the equation of the tangent to the curve $y = \dfrac{6}{2x-1}$ at the point (2, 2).

4. Find the equation of the normal to the curve $y = \sqrt{5x^2+16}$ at the point (2, 6).

5. A semicircle has equation $x^2 + y^2 = 25$, where $y \geq 0$.
   (i) Find y in terms of x.
   (ii) Find $\dfrac{dy}{dx}$.
   (iii) Deduce the gradient of the curve at the point (4, 3).



6. An ice cube is melting, and at time t hours it has the form of a cube of side x cm, and its volume is V cm^3. At a certain instant, each edge measures 20 cm and is decreasing at 0.2 cm h^{-1}. At what rate is the volume decreasing?

7. Find where the tangent to the curve $y = \dfrac{10}{\sqrt{x^2+9}}$ at the point where $x = 4$ meets both axes.

8. The gradient of $y = \dfrac{12}{6-x}$ at the point P is 3. Work out the possible coordinates of P.

9. A balloon is modelled as a sphere. It is inflated at a rate of 30 cm³ per second. At the point when the radius is 10 cm, what is the rate of increase of the radius?

10. A curve has equation $y = \sqrt{7x-3}$. The normal to the curve at the point where $x = 4$ meets the x-axis at the point P. Work out the coordinates of the point P.