Section 1: Differentiating exponentials and logarithms

Exercise level 3

1. (i) Express 2^x in the form $\mathrm{e}^{x\ln a}$, for a suitable constant a.

   (ii) Hence, using the chain rule, find the derivative of $y = 2^x$, expressing $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a multiple of 2^x.

   (iii) Express $y = x^x$ as an exponential, and hence find the derivative of $y = x^x$, $x > 0$. Also find the exact coordinates of the stationary point on the curve.

2. (i) Given $y = \ln x$, express x as a function of y and hence prove that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$.

   (ii) Given $y = \ln(\ln x)$, use the chain rule to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

   (iii) By first simplifying $y = \ln(\ln x^x)$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\ln(\mathrm{e}x)}{x\ln x}$.

3. (i) Given that $\mathrm{f}(x) = \ln(1+x) - x + \frac{1}{2}x^2$, $x \geq 0$, find $\mathrm{f}'(x)$.

   (ii) Show that $\mathrm{f}'(x) > 0$ for $x > 0$, and deduce that $\ln(1+x) > x - \frac{1}{2}x^2$, $x > 0$.

   (iii) Similarly show that $\ln(1+x) < x - \frac{1}{2}x^2 + \frac{1}{3}x^3$, $x > 0$.

   (iv) Hence show that $\frac{3}{8} < \ln\frac{3}{2} < \frac{5}{12}$.