Section 1: Differentiating exponentials and logarithms

Exercise level 2

1. If $f(x) = (1+x^2)e^{3x}$, find $f'(x)$.

2. If $y = e^{x^2} \ln x$, find $\frac{dy}{dx}$.

3. (i) Show that the derivative of 2^x is $2^x \ln 2$.
   (ii) Hence find the derivative of 2^{3x}.

4. Find the exact coordinates of the turning point of the curve $y = x \ln x$.
   By finding $\frac{d^2y}{dx^2}$, determine its nature.

5. Find the exact coordinates of the turning point of the curve $y = xe^{-2x}$.
   By finding $\frac{d^2y}{dx^2}$, determine its nature.

6. Given that $y = \frac{e^x}{\sqrt{1+2x}}$, show that $\frac{dy}{dx} = \frac{2xe^x}{\sqrt{(1+2x)^3}}$.

7. The temperature T°C of the water in a kettle t minutes after boiling is modelled by the equation $T = 20 + 80e^{-0.5t}$. Find the initial rate of cooling, and the rate of cooling after 2 minutes.