Section 2: Differentiating trigonometric functions

Exercise level 3

1. (i) Given that $f(x) = \sin^2 x + \cos^2 x$, show that $f'(x) = 0$ for all x.

   (ii) By considering $f'(0)$, deduce a well-known identity.

2. (i) Given $y = \arcsin x$, express x in terms of y, and hence find $\dfrac{dx}{dy}$ as a function of y.

   (ii) Express $\dfrac{dy}{dx}$ as a function of x and hence find $\displaystyle\int \dfrac{1}{\sqrt{1-x^2}}\, dx$.

3. (i) Find $\dfrac{dy}{dx}$ for the function $y = \cos(\pi e^x)$.

   (ii) Show that the stationary points occur when $x = \ln m$, where m is a non-negative integer.

   (iii) Show that the distance between successive stationary points tends to 0 as $m \to \infty$.

   (iv) Sketch the graph of $y = \cos(\pi e^x)$.