

Section 2: Differentiating trigonometric functions

Exercise level 2

1. If $f(x) = e^{2x} \tan x$, find $f'(x)$.
2. Show that $y = \sin x + 2 \cos x$ has a stationary point when $\tan x = \frac{1}{2}$.
Given that $0 < x < \frac{\pi}{2}$, find the exact value of y at this point.
[Hint: sketch a right angled triangle with short sides 1 and 2, and use this to find $\sin x$ and $\cos x$ given that $\tan x = \frac{1}{2}$]
3. Given that $y = \sin 3x$, show that $\frac{d^2 y}{dx^2} = -9y$.
4. Differentiate $\sin^3 x$. Use your result to evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$.
5. The height x metres of water above a sand-bar t hours after noon on a certain day is modelled by the equation $x = 5 \sin \frac{1}{2} t$.
 - (i) Calculate the height of the water at 2 p.m., and the approximate rate at which the water is rising at this time.
 - (ii) When is the water rising most rapidly, and at what rate?
6. A weight on the end of a spring, which is hanging vertically, is oscillating so that its height y metres above the floor after t seconds is given by:
$$y = 0.5 + 0.2 \sin(10t).$$
 - (i) What are the greatest and least heights of the weight?
 - (ii) Find the speed of the weight after 0.5 seconds.