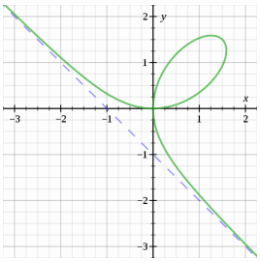
Section 1: Using parametric equations

Exercise level 3

1. The curve C has parametric equations
$x = \mathrm{f}(t)$ and $y = \mathrm{g}(t)$,
where $\mathrm{f}(t) = 2t^2 - 2$ and $\mathrm{g}(t) = t^3 - t$.
   (i) Sketch the graphs of $x = \mathrm{f}(t)$ and $y = \mathrm{g}(t)$ and hence find the range of values for x and y in the curve C.
   (ii) Find the points where the curve C meets the line $y = x$.
   (iii) Show that the Cartesian equation of the curve C is $8y^2 = x^3 + 2x^2$. Use graphing software to sketch the curve.

2. The folium of Descartes (the Latin word *folium* means leaf) is defined by the equation
$x^3 + y^3 = 3xy$.
The curve is shown in the diagram below.



Folium of Descartes

   (i) Show that the curve is symmetric in the line $y = x$.
   (ii) By using the substitution $y = tx$, show that the parametric equations of the curve are
$$x = \frac{3t}{1+t^3} \text{ and } y = \frac{3t^2}{1+t^3}.$$
   (iii) Find the points where the curve meets the line $y = x$.
   (iv) By using implicit differentiation, show that the maximum point on the leaf (first quadrant) corresponds to $t = 2^{\frac{1}{3}}$ and find the coordinates of the maximum point.

3. A curve has parametric equations $x = \cos^2 t$, $y = \frac{1}{2}\sin 2t$, where $0 \le y < \pi$.
   (i) Show that the curve is a circle and find its Cartesian equation. Sketch the curve.
   (ii) Indicate in your diagram the points on the circle for $t = 0$, $t = \frac{\pi}{4}$ and $t = \frac{\pi}{2}$.
   (iii) Given that θ is the angle at the centre of the circle between the x-axis and the line joining the centre and a point P on the circle, show that $\tan\theta = \tan 2t$ for $0 < t < \frac{\pi}{4}$ and $\tan\theta = -\tan 2t$ for $\frac{\pi}{4} < t < \frac{\pi}{2}$.