Section 2: Further trigonometric equations

Exercise level 3

1. Find the points of intersection of the curves $y = \cos x$ and $y = 2 + \sqrt{3}\sin x$ for $0 < x < 9\pi$.

2. Show that $x = \frac{\pi}{4}$ is a root of the equation
$$2\sqrt{5}\sin x + 4\sqrt{5}\cos x = 3\sqrt{10}.$$
Show further that $2\sqrt{5}\sin x + 4\sqrt{5}\cos x = R\sin(x + \theta)$, where R is a constant to be determined and $\theta = \arctan 2$. Hence, or otherwise, show that
$$\pi = 4\arcsin\left(\frac{3}{\sqrt{10}}\right) - 4\arctan 2.$$

3. The function $\mathrm{g}(x)$ is defined by
$$\mathrm{g}(x) = 7\cos^2 x + \sin^2 x - 8\sin x\cos x$$
(i) Show that $\mathrm{g}(x)$ can be expressed in the form $a + b\cos(2x + \alpha)$ where $\tan\alpha = \frac{4}{3}$ and a, b are constants to be determined. Find the greatest and the least values of $\mathrm{g}(x)$.
(ii) Find, in terms of α, the least positive value of x for which $\mathrm{g}(x) = 0$.

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