Section 2: Further trigonometric equations

Exercise level 2

1. Express $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ where $R > 0$ and α is acute, and hence find the maximum and minimum values of the function.

2. Write each of the following functions in the form $R\cos(\theta \pm \alpha)$, where $R > 0$ and α is acute.
   (i) $\cos\theta + \sin\theta$
   (ii) $2\cos\theta - \sin\theta$
   (iii) $\sqrt{3}\sin\theta + \cos\theta$

3. Express $5\sin\theta - 8\cos\theta$ in the form $r\sin(\theta - \alpha)$ given that $r > 0$ and α is acute, and hence solve the equation $5\sin\theta - 8\cos\theta = 6$ for $0° \leq \theta \leq 360°$.

4. Express $7\sin x + 24\cos x$ in the form $r\sin(x + \alpha)$ given that α is acute and $r > 0$. Hence find the maximum and minimum values of the function and the values of x where they occur, for $0 < x < 2\pi$.