

Section 1: The compound angle identities

Exercise level 3

- Show that $\left[1 - \cos\left(\theta + \frac{\pi}{4}\right)\right]\left[1 + \sin\left(\theta + \frac{\pi}{4}\right)\right] = \left(\frac{1}{\sqrt{2}} + \sin\theta\right)^2$. Hence find the values of k for which the equation $\left[1 - \cos\left(\theta + \frac{\pi}{4}\right)\right]\left[1 + \sin\left(\theta + \frac{\pi}{4}\right)\right] = k$ has no real roots.
- Prove the identity $\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$. Hence find $\tan \theta$ given that $\tan 3\theta = 1$ and $0 < \theta < \frac{\pi}{4}$.
- Let $f(x) = \sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x$.

 - Show that $\sin 3x = \sin x(4 \cos^2 x - 1)$.
 - Show that there are no values of x in the interval $0 < x < \pi$ for which $f(x) = 0$.