

Section 1: Trigonometric functions and identities

Exercise level 2

1. Simplify the following expressions.

(i) $\frac{1 - \sec^2 x}{1 - \operatorname{cosec}^2 x}$

(ii) $\frac{1 - \sin^2 x}{1 + \tan^2 x}$

(iii) $\frac{\sin x}{1 + \cot^2 x}$

2. Solve the following equations for $0^\circ \leq x \leq 180^\circ$.

(i) $\operatorname{cosec}(x + 10^\circ) = 3$

(ii) $\cot(x - 30^\circ) = 0.45$

3. For each of the following functions:

(a) state the range of the function

(b) find the inverse of the function, and give its domain

(c) sketch the function and its inverse on the same graph.

(i) $f(x) = 1 + \sin 2x$ $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

(ii) $f(x) = \frac{1}{2} \tan(x - \frac{\pi}{2})$ $0 \leq x \leq \pi$

4. Prove the following identities.

(i) $\tan^2 \theta + \cot^2 \theta \equiv \sec^2 \theta + \operatorname{cosec}^2 \theta - 2$

(ii) $\sec^2 \theta \equiv \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \sin \theta}$

(iii) $(1 - \cos \theta)(1 + \sec \theta) \equiv \sin \theta \tan \theta$

5. Solve the following equations for $0^\circ \leq x \leq 360^\circ$.

(i) $2 \tan x \operatorname{cosec} x = 3$

(ii) $\tan^2 x = \sec x \tan x$

6. Solve the following for $-180^\circ \leq x \leq 180^\circ$.

(i) $2 \tan x - \cot x = 1$

(ii) $2 \cos x = 3 - \sec x$

(iii) $\tan^2 x = \sec x + 5$