

Section 1: The general binomial expansion**Exercise level 3 (Extension)**

1. Find the sum to infinity of the series

$$1 - x + \frac{1 \times 3}{1 \times 2} x^2 - \frac{1 \times 3 \times 5}{1 \times 2 \times 3} x^3 + \dots$$

Hint: compare coefficients with those in the expansion of $(1 + ax)^n$.

2. Show that for small x , $\frac{1}{1-x} - \frac{1}{1+x} \approx 2x$, and deduce an approximation for

$$\frac{1000}{0.999} - \frac{1000}{1.001}.$$

3. Show that the coefficient of x^{2n} in the expansion of $(1 + \sqrt{2}x)^{-2}$ is $(2n+1)2^n$.
4. Expand $(2+x)^{-2}$ in ascending powers of $\frac{1}{x}$, giving the range of values for which the expansion is valid.
5. Given that the coefficients of x^2 and x^3 are equal in the expansion of $(1+nx)^n$, find the possible values of n .