Section 2: Arithmetic sequences and series

Exercise level 3 (Extension)

1. Fred has recently bought a new bathroom suite, at a cost of £2000 in total, including any interest payments. He chooses to pay using the supplier's 'easy terms' Scheme A.

   A: Pay £20 in the first month and then increase the payments by £4 every subsequent month.

   (i) Explain why in scheme A, Fred's payments would form an arithmetic sequence.

   (ii) Find two formulae for u_n, the figure Fred pays in the n^{th} month, and S_n, the total that Fred has paid after n months.

   (iii) Use your formulae in (ii) to determine in which month Fred has finally fully paid for his purchase.

   (iv) In fact, the final month's payment is too much to cover the remaining debt. Calculate how much Fred should pay in the final month to clear his debt exactly.

2. Priya has recently left university and started a job, so she is beginning to save money for a pension. She expects to save £1000 from her earnings in the first year, and then increase her annual savings by £200 so that in year 2 she will save £1200, in year 3 £1400, and so on. She intends to follow this plan for the next 30 years.

   (i) How much does she expect to save in total?

   (ii) She decides that she needs to save more, and sets a 'pension pot' of £150000 as her target. She feels she cannot save more in the first year, so what annual increase must she plan to make instead?

   (iii) In fact, after 10 years at work (using the scheme in part (ii) above) she finds that she earns enough to modify her plans. She decides to increase her payment in year 11 and all subsequent years by £350 each year instead of £200. What will now be her total savings after 30 years?

3. Jane sets out on a 'gap year' trip to the Pacific with £13000 saved in her account. She pays £450 for her initial airfare, and requires a further £500 for her return fare at the end of the year.

   (i) She expects her expenditure to increase by £100 per month while she is away, and uses her knowledge of arithmetic series to budget. What can she plan to spend in month one, if her trip lasts exactly 12 months?

   (ii) After 5 months, she makes an unexpected return to the UK to attend a friend's wedding, which costs her £1000. If she then changes her budget, and wants to continue to increase her monthly expenditure by an adjusted £75, how much can she now plan to spend in month 6?