Section 1: Definitions and notation

Exercise level 3 (Extension)

1. Fred is given a choice of 2 repayment schemes for some furniture he has recently bought.
   A: Pay £20 in the first month and then increase the payments by £4 every subsequent month.
   B: Pay £20 in the first month, followed by an increase of 12% (to the nearest penny) in every subsequent month.

   (i) Explain whether in either or both of schemes A and B, Fred's payments would form an arithmetic or a geometric sequence.

   (ii) Fred starts to produce the following table to calculate his payments in the first 5 months:

month	scheme A		scheme B	
	payment	total	payment	total
1	20	20	20	20
2	24	44	22.40	42.40
3	28	72	25.09	67.49
4				
5				

   Complete the table for the first 5 months.

   (iii) Fred owes £1000 for his new furniture. By extending the table (or by calculation, for which you are likely to need to know about logarithms) find which payment scheme pays off his debt earlier.

   (iv) From your extended table (or by direct calculation) find in which month the total payment by both schemes is close to being equal.

2. "Bumper bees" have never been recorded in Britain until recently. A colony of bumper bees will, normally, create another new colony in each of the first and second seasons of its existence. But the colony will itself die out before its third season.
   For the first time ever, a single colony was reported in Britain in 2001, which produced, as expected, an extra new colony in 2002 and 2003, though it didn't survive to produce any further colonies.

   (i) Complete the second row in the table below, to show the number of new colonies that would have appeared in the years up to 2015, if there were no predators present:

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
new colonies	1	1													
total colonies	1	2													
ratio y_{n+1}/y_n	-----	1													

(ii) Complete the third row in the table, to show the total number of live colonies in any year.

(iii) Complete the final row, to show the ratio of the number of live colonies in any two consecutive years (this shows the 'growth rate' of the total number of colonies). What happens to the ratio?

(iv) Use your results from (iii), based on the final column for 2015, to make a (very) rough estimate of the number of live colonies likely in 2020, and compare it with a calculated figure by extending the table.