Section 1: Functions, graphs and transformations

Exercise level 3 (Extension)

1. Find the range of the function $\mathrm{f}(x) = x^2 - 2x + 4$ with domain $-1 \le x \le 2$.

2. Consider the function $\mathrm{f}(x) = \dfrac{2x}{x^2+1}$

   (i) By letting $y = \mathrm{f}(x)$, show that $yx^2 - 2x + y = 0$.

   (ii) Using the discriminant of a quadratic, find the range of values for y such that the equation in (i) has real roots.

   (iii) Deduce the range of the function $\mathrm{f}(x)$ and hence sketch the graph of $y = \mathrm{f}(x)$.

3. The point P on the curve $y = \mathrm{f}(x)$ is transformed to point P′ $(6, 8)$ under the transformation $y = 2\mathrm{f}(x-4)$. Find the coordinates of P.

4. Describe two transformations, in the order specified (**T** for translation, **S** for stretch), taking the original curve to the transformed curve:

Original	transformed	order
$y = x^2$	$y = 2x^2 + 6$	**T** then **S**
$y = \sqrt{x}$	$y = \sqrt{4x+2}$	**S** then **T**
$y = \sqrt{x}$	$y = \sqrt{4x} + 2$	**T** then **S** (in *y*-direction)
$y = x^2$	$y = 4x^2 - 4x + 1$	**S** (in *x*-direction) then **T**