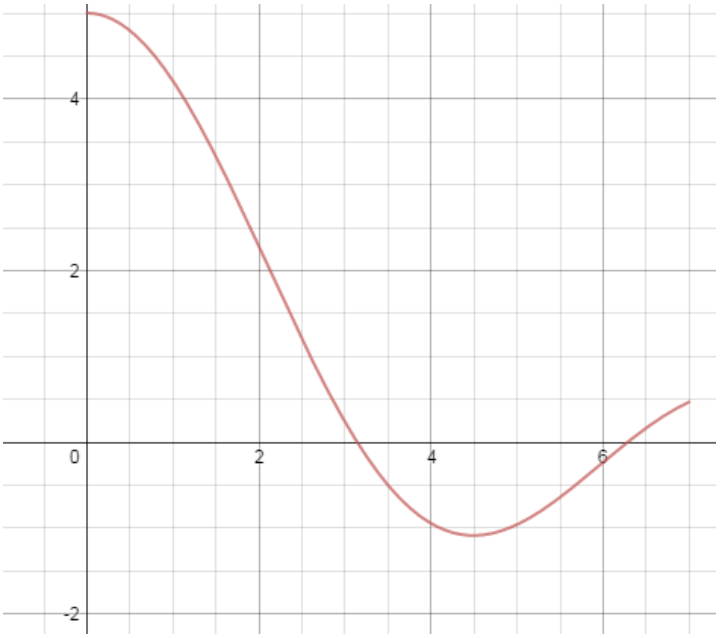
Section 1: Functions, graphs and transformations

Exercise level 2

1. The function d is defined as: d(n) = the number of factors of n, where $n \in \mathbb{Z}$.
   E.g. d(25) = 3 since 25 has 3 factors: 1, 5 and 25.
   (i) Find (a) d(6) (b) d(5) (c) d(16) (d) d(13)
   (ii) Describe the set of numbers for which d(n) = 2
   (iii) Describe the set of numbers for which d(n) is an odd number.

2. The function f is defined as:
   $$\text{f}: x \to \frac{1}{x-1}.$$
   (i) What value of x must be excluded from the domain of this function?
   (ii) Find (a) f(2) (b) f(– 3) (c) f(0)
   (iii) For what value of x is f(x) = 2?

3. The diagram below shows the graph of a function f(x), where $0 \le x \le 7$.
   The graph passes through the point A(0, 5), B(π, 0) and C(2π, 0) and has a turning point at D(4.5, -1.1).



   Sketch the following graphs, giving the coordinates of the turning point, D, and the images of A, B and C in each case.
   (i) y = f(x – 2) + 0.5
   (ii) y = f($\frac{1}{2}x$ + 3)
   (iii) Write down the new domain and range in each case.

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4. The graph $y = x^2 - 2x$ undergoes the following transformations.

   Translation through $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, followed by a stretch parallel to the *x*-axis, scale factor $\frac{1}{3}$ and a reflection in the *y*-axis.

   (i) Find the equation of the new graph in the form $y = ax^2 + bx + c$

   (ii) Write down the equation of the line of symmetry of the final graph.

   (iii) Write down the coordinates of the turning point of the new graph.

5. Explain how you could use transformations to obtain each of the following graphs from the graph of $y = \sin x$.

   (i) $y = 1 + \sin(x + 30°)$

   (ii) $y = -3\sin \frac{1}{2}x$

6. (i) Write $y = 5x^2 - 15x + 4$ in completed square form.

   (ii) Write down the sequence of transformations that should be done to $y = 5x^2 - 15x + 4$ to map it onto $y = x^2$.

   (iii) Show algebraically that the sequence of transformations will map $y = 5x^2 - 15x + 4$ onto $y = x^2$.