Section 2: Composite and inverse functions

Exercise level 3

1. Given $\mathrm{f}(x) = x + 1$ and $\mathrm{g}(x) = \sqrt{x - 1}$, find the missing numbers in the brackets in the following composite functions:

$\mathrm{ff}(\quad) = 1$	$\mathrm{gf}(\quad) = 1$	$\mathrm{fg}(\quad) = 1$	$\mathrm{gg}(\quad) = 1$
$\mathrm{ff}(\quad) = 2$	$\mathrm{gf}(\quad) = 2$	$\mathrm{fg}(\quad) = 2$	$\mathrm{gg}(\quad) = 2$
$\mathrm{ff}(\quad) = 3$	$\mathrm{gf}(\quad) = 3$	$\mathrm{fg}(\quad) = 3$	$\mathrm{gg}(\quad) = 3$
$\mathrm{ff}(\quad) = 4$	$\mathrm{gf}(\quad) = 4$	$\mathrm{fg}(\quad) = 4$	$\mathrm{gg}(\quad) = 4$
$\mathrm{ff}(\quad) = 5$	$\mathrm{gf}(\quad) = 5$	$\mathrm{fg}(\quad) = 5$	$\mathrm{gg}(\quad) = 5$

2. Express each of the following functions as suitable compositions of

$$\mathrm{f}(x) = 4^x, \qquad \mathrm{g}(x) = \sqrt{x}, \qquad \mathrm{h}(x) = \frac{1}{x}, \qquad \mathrm{j}(x) = 4x$$

(i) x

(ii) $2\sqrt{x}$

(iii) 4^{x+1}

(iv) 2^x

(v) $8\sqrt{x}$

3. The function $\mathrm{f}(x) = ax^2 + b, \quad x \geq 0$, satisfies $\mathrm{f}^{-1}(1) = 1$ and $\mathrm{f}^{-1}(2) = 2$. Find the value of $\mathrm{f}^{-1}(3)$.

4. (i) Find the largest integer k such that the function $\mathrm{f}(x) = x^2 + 4x + 3$ with (restricted) domain $x \leq k$, is a one-to-one function.

(ii) Find an expression for $\mathrm{f}^{-1}(x)$.

(iii) State the geometrical relationship between the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$.

(iv) Show algebraically that the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ do not meet.