## Section 1: Methods of proof

Exercise level 1

1. For each of the following conjectures, decide whether it is true or false. If it is true prove it, stating which method you have used. If it is false give a counter-example.
    (i)   If a is a factor of b and a is a factor of c then a is a factor of $b + c$, where a, b and c are positive integers.
    (ii) 103 is a prime number
    (iii) $n^2 + n + 1$ is a prime number for all values of n.
    (iv) If a and b are real numbers, then $a^2 + b^2 \geq 2ab$.
    (v)   The cube root of 2 is irrational.

2. (i)   Prove that if a and b are rational numbers, then $a + b$ is a rational number.
    (ii) Give a counter-example to disprove the statement that if $a + b$ is a rational number, then a and b are both rational numbers.
    (iii) Use the method of proof by contradiction to prove that if a is a rational number and b is an irrational number, then $a + b$ is an irrational number.

3. The triangle below has a right-angle at C.
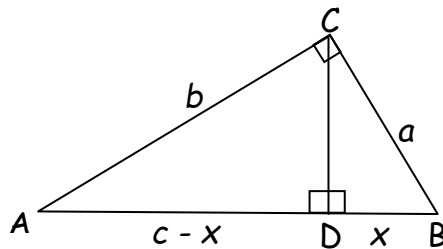
    (i)   Explain why triangle ADC is similar to triangle ACB.
          Use these similar triangles to write down a relationship between a, b, c and x.
    (ii) Explain why triangle BDC is similar to triangle BCA.
          Use these similar triangles to write down another relationship between a, b, c and x.
    (iii) Eliminate x from the two equations you have found.
          What well-known result have you proved?

4. Prove that there are no positive integer solutions to the equation $x^2 - y^2 = 1$ using the method of proof by contradiction as follows:
    (i)   Write the left hand side of the equation as the product of two factors.
    (ii) Assume that a solution (x, y) exists where x and y are positive integers.
    (iii) Write down the possible values of each factor of the left-hand side.
    (iv) Show that each case leads to values of x and y which are not both positive integers.

5. Use the method of Question 4 to show that there are no positive integer solutions to the equation $x^2 - y^2 = 10$.