Section 2: The vector equation of a line

Section test

1. The vector equation of the line through the point A $(2,3)$ in the direction of the vector $\begin{pmatrix}2\\4\end{pmatrix}$ can be written as:

(a) $\mathbf{r}=\begin{pmatrix}2\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\3\end{pmatrix}$

(b) $\mathbf{r}=\begin{pmatrix}0\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\2\end{pmatrix}$

(c) $\mathbf{r}=\lambda\begin{pmatrix}2\\4\end{pmatrix}$

(d) $y=2x-1$

2. Find the coordinates of the point of intersection of the lines

$$\mathbf{r}=\begin{pmatrix}-2\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\2\end{pmatrix}$$

and $$\mathbf{r}=\begin{pmatrix}-1\\-2\end{pmatrix}+\mu\begin{pmatrix}3\\-1\end{pmatrix}$$

3. Find the angle between the lines

$$\mathbf{r}=\begin{pmatrix}2\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\end{pmatrix} \text{ and } \mathbf{r}=\begin{pmatrix}1\\-2\end{pmatrix}+\mu\begin{pmatrix}1\\3\end{pmatrix}.$$

4. The cartesian equation of the line $\mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}-1\\0\\2\end{pmatrix}$ is

(a) $x-2=\frac{z-3}{2}$ and $y=1$

(b) $2-x=\frac{z-3}{2}$ and $y=1$

(c) $2-x=\frac{z-3}{2}=y-1$

(d) $x-2=\frac{z-3}{2}=y-1$

5. The vector equation of the line through the points A(-1, 2, 3) and B(2, 1, 4) is

(a) $\mathbf{r}=\begin{pmatrix}2\\1\\4\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}$

(b) $\mathbf{r}=\begin{pmatrix}-1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\4\end{pmatrix}$

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(c) $\mathbf{r}=\begin{pmatrix}-1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\3\\7\end{pmatrix}$ (d) $\mathbf{r}=\begin{pmatrix}-1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\1\end{pmatrix}$

6. The vector equation of the line $\frac{x-2}{3}=\frac{y}{2}=z+1$ is

(a) $\mathbf{r}=\begin{pmatrix}3\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\0\\-1\end{pmatrix}$ (b) $\mathbf{r}=\begin{pmatrix}2\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\2\\1\end{pmatrix}$

(c) $\mathbf{r}=\begin{pmatrix}2\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\2\\0\end{pmatrix}$ (d) $\mathbf{r}=\begin{pmatrix}3\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\0\\-1\end{pmatrix}$

7. Find the angle between the lines $\mathbf{r}=\begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\2\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}4\\1\\-2\end{pmatrix}+\mu\begin{pmatrix}2\\0\\-1\end{pmatrix}$.

8. Find the angle between the line $\mathbf{r}=\begin{pmatrix}1\\0\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}$ and the z-axis.

9. Find the coordinates of the point of intersection of

$l_1:\mathbf{r}=\begin{pmatrix}2\\1\\5\end{pmatrix}+\lambda\begin{pmatrix}-1\\3\\2\end{pmatrix}$ and $l_2:\mathbf{r}=\begin{pmatrix}5\\-4\\3\end{pmatrix}+\mu\begin{pmatrix}-5\\13\\8\end{pmatrix}$.

10. The line L is $\frac{x-3}{1}=\frac{y}{-1}=\frac{z-1}{2}$

The line M is $\frac{x+2}{2}=\frac{y-3}{-1}=\frac{z+1}{1}$

The line N is $\frac{x-4}{1}=\frac{y-1}{-3}=\frac{z-3}{2}$

The relationships between the line L and each of the lines M and N is

(a) L and M meet, L and N are skew (b) L and M are skew, L and N meet

(c) L meets both M and N (d) L and M are skew, L and N are skew

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Section test solutions

1. $\underset{\sim}{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and the line is in the direction $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda(\underset{\sim}{b} - \underset{\sim}{a}) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

2. The intersection of these lines is at $\begin{pmatrix} -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$$x: -2 + \lambda = -1 + 3\mu \Rightarrow \lambda - 3\mu = 1$$

$$y: 3 + 2\lambda = -2 - \mu \Rightarrow 2\lambda + \mu = -5$$

Solving simultaneously: $2\lambda - 6\mu = 2$

$$2\lambda + \mu = -5$$

$$7\mu = -7$$

$$\mu = -1$$

Putting this value back into $\underset{\sim}{r} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \end{pmatrix}$:

$$\underset{\sim}{r} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} - 1 \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$$

The point of intersection has coordinates $(-4, -1)$.

3. Using the direction vectors:

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = (-1) \times 1 + 2 \times 3 = 5$$

$$|\underset{\sim}{a}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$$

$$|\underset{\sim}{b}| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \frac{5}{\sqrt{5}\sqrt{5 \times 2}} = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

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4. Using $\dfrac{x-a_1}{u_1}=\dfrac{y-a_2}{u_2}=\dfrac{z-a_3}{u_3}$

$$\frac{x-2}{-1}=\frac{z-3}{2} \text{ and } y-1=0$$

$$2-x=\frac{z-3}{2} \text{ and } y=1$$

5. Direction vector is $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}2\\1\\4\end{pmatrix}-\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\-1\\1\end{pmatrix}$

Equation of line is $\underset{\sim}{r}=\begin{pmatrix}-1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\1\end{pmatrix}$

6. Writing the equation in the form $\dfrac{x-a_1}{u_1}=\dfrac{y-a_2}{u_2}=\dfrac{z-a_3}{u_3}$

$$\frac{x-2}{3}=\frac{y}{2}=z+1 \Rightarrow \frac{x-2}{3}=\frac{y-0}{2}=\frac{z-(-1)}{1}$$

In vector form this is $\underset{\sim}{r}=\begin{pmatrix}2\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\2\\1\end{pmatrix}$

7. Using the direction vectors: $\cos\theta=\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$

$$\underset{\sim}{a}\cdot\underset{\sim}{b}=\begin{pmatrix}3\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}2\\0\\-1\end{pmatrix}=3\times2+(-1)\times0+2\times(-1)=4$$

$$|\underset{\sim}{a}|=\sqrt{3^2+(-1)^2+2^2}=\sqrt{14}$$

$$|\underset{\sim}{b}|=\sqrt{2^2+0^2+(-1)^2}=\sqrt{5}$$

$$\cos\theta=\frac{4}{\sqrt{14}\sqrt{5}}$$

$$\theta=61.4^\circ \text{ (1 d.p)}$$

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8. The direction vector is $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

The direction vector for the z-axis is $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

The angle between the line and the z-axis is given by

$$\cos\theta = \frac{\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{2^2 + 1^2 + 3^3}\sqrt{1^2}}$$

$$\cos\theta = \left(\frac{3}{\sqrt{14}}\right)$$

$$\theta = 36.7^\circ$$

9. The coordinates of points on l_1 are $\left(2-\lambda, 1+3\lambda, 5+2\lambda\right)$.

The coordinates of points on l_2 are $\left(5-5\mu, -4+13\mu, 3+8\mu\right)$.

At the point of intersection,

$$2-\lambda = 5-5\mu \qquad -\lambda + 5\mu = 3 \quad ①$$
$$1+3\lambda = -4+13\mu \Rightarrow 3\lambda - 13\mu = -5 \quad ②$$
$$5+2\lambda = 3+8\mu \qquad \lambda - 4\mu = -1 \quad ③$$

① + ③ ⇒ $\mu = 2, \lambda = 7$

These values also satisfy equation ②.

The coordinates of the point of intersection are (-5, 22, 19).

10. Points on L have coordinates $\left(3+\lambda, -\lambda, 1+2\lambda\right)$

Points on M have coordinates $\left(-2+2\mu, 3-3\mu, -1+\mu\right)$

Points on N have coordinates $\left(4+t, 1-3t, 3+2t\right)$

If L and M meet,

$$3+\lambda = -2+2\mu \qquad \lambda - 2\mu = -5 \quad ①$$
$$-\lambda = 3-3\mu \Rightarrow -\lambda + 3\mu = 3 \quad ②$$
$$1+2\lambda = -1+\mu \qquad 2\lambda - \mu = -2 \quad ③$$

① + ② ⇒ $\mu = -2, \lambda = -9$

These values do not satisfy equation ③, so lines L and M are skew.

If L and N meet,

$3 + \lambda = 4 + t \qquad \lambda - t = 1$ ①

$-\lambda = 1 - 3t \Rightarrow -\lambda + 3t = 1$ ②

$1 + 2\lambda = 3 + 2t \qquad \lambda - t = 1$ ③

① + ② $\Rightarrow 2t = 2$

$\Rightarrow t = 1, \lambda = 2$

These values also satisfy equation ③, so lines L and N meet.