Section 1: The scalar product

Section test

1. Find the value of $\begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

2. Find the value of $\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$.

3. The angle between two vectors is 90º.
   What is the scalar product of the two vectors?

4. Find the acute angle between the vectors $5\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $-2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$.

5. Which of the following vectors are perpendicular to the vector $4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$?
   $\mathbf{i} - \mathbf{j} - \mathbf{k}$
   $-2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$
   $-\mathbf{i} - 4\mathbf{k}$

6. The vector $\begin{pmatrix} 3 \\ k \\ 2k-1 \end{pmatrix}$ is perpendicular to the vector $\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$.
   Find the value of *k*.

7. The vector $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ is perpendicular to vector $\mathbf{b}$.
   The vector $\mathbf{b}$ could be equal to

(a) $4\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$
(b) $-3\mathbf{i} - 4\mathbf{j}$
(c) $-3\mathbf{i} + 4\mathbf{j}$
(d) $-4\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

8. The points P, Q and R have coordinates (0, -1, 4), (2, -1, 3) and (-1, 2, 0) respectively.
   Find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.
   Find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.
   The angle you have just calculated is

(a) $180° - \angle QPR$ (b) $\angle QPR$

(c) $180° - \angle PQR$ (d) $\angle PQR$

Edexcel AS FM Vectors 1 section test solutions

Section test solutions

1. $\begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} = (2 \times 1) + (-3 \times 4)$

$= 2 - 12$

$= -10$

2. $\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = (-1) \times 2 + 2 \times 1 + (-3) \times 0 = 0$

3. When two lines are perpendicular to each other then

$\theta = 90^\circ \Rightarrow \cos\theta = 0 \Rightarrow \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = 0$

If $\frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = 0$ then $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

4. $\underset{\sim}{a} \cdot \underset{\sim}{b} = \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix} . \begin{pmatrix} -2 \\ 3 \\ 3 \end{pmatrix} = (5 \times -2) + (-1 \times 3) + (2 \times 3) = -7$

$|\underset{\sim}{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{30}$

$|\underset{\sim}{b}| = \sqrt{(-2)^2 + 3^2 + 3^2} = \sqrt{22}$

$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \frac{-7}{\sqrt{30}\sqrt{22}}$

$\theta = 105.8^\circ$

The acute angle between the vectors is 74.2° (1 d.p.)

5. If they are perpendicular, the scalar product is zero

$\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = (4 \times 1) + (3 \times -1) + (-1 \times -1) = 4 - 3 + 1 = 2$

$\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} = (4 \times -2) + (3 \times 3) + (-1 \times 1) = -8 + 9 - 1 = 0$ so perpendicular

$$\begin{pmatrix}4\\3\\-1\end{pmatrix}\cdot\begin{pmatrix}-1\\0\\-4\end{pmatrix}=(4\times-1)+(3\times0)+(-1\times-4)=-4+0+4=0 \text{ so perpendicular}$$

6. $$\begin{pmatrix}3\\k\\2k-1\end{pmatrix}.\begin{pmatrix}2\\-3\\-1\end{pmatrix}=0$$

$6-3k-(2k-1)=0$

$6-3k-2k+1=0$

$5k=7$

$k=1.4$

7. If they are perpendicular, $\underset{\sim}{a}\cdot\underset{\sim}{b}=0$

$$\underset{\sim}{a}\cdot\underset{\sim}{b}=\begin{pmatrix}4\\3\\1\end{pmatrix}.\begin{pmatrix}4\\-3\\-7\end{pmatrix}=(4\times4)+(3\times-3)+(1\times-7)=0$$

$$\underset{\sim}{a}\cdot\underset{\sim}{b}=\begin{pmatrix}4\\3\\1\end{pmatrix}.\begin{pmatrix}-3\\-4\\0\end{pmatrix}=(4\times-3)+(3\times-4)+(1\times0)=-24\neq0$$

$$\underset{\sim}{a}\cdot\underset{\sim}{b}=\begin{pmatrix}4\\3\\1\end{pmatrix}.\begin{pmatrix}-3\\4\\0\end{pmatrix}=(4\times-3)+(3\times4)+(1\times0)=0$$

$$\underset{\sim}{a}\cdot\underset{\sim}{b}=\begin{pmatrix}4\\3\\1\end{pmatrix}.\begin{pmatrix}-4\\3\\-7\end{pmatrix}=(4\times-4)+(3\times3)+(1\times-7)=-14\neq0$$

The vector $\underset{\sim}{b}$ could be $4\underset{\sim}{i}-3\underset{\sim}{j}-7\underset{\sim}{k}$ or $-3\underset{\sim}{i}+4\underset{\sim}{j}$

8. $$\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=\begin{pmatrix}2\\-1\\3\end{pmatrix}-\begin{pmatrix}0\\-1\\4\end{pmatrix}=\begin{pmatrix}2\\0\\-1\end{pmatrix}$$

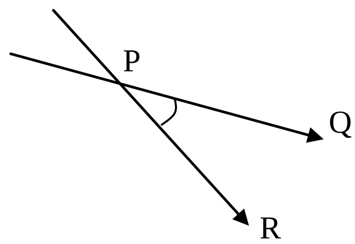
$$\overrightarrow{PR}=\overrightarrow{OR}-\overrightarrow{OP}=\begin{pmatrix}-1\\2\\0\end{pmatrix}-\begin{pmatrix}0\\-1\\4\end{pmatrix}=\begin{pmatrix}-1\\3\\-4\end{pmatrix}$$

$$\overrightarrow{PQ}.\overrightarrow{PR} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} . \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} = -2 + 0 + 4 = 2$$

$$\left|\overrightarrow{PQ}\right| = \sqrt{2^2 + 0^2 + (-1)^2} = \sqrt{5}$$

$$\left|\overrightarrow{PR}\right| = \sqrt{(-1)^2 + 3^3 + (-4)^2} = \sqrt{26}$$

Angle between vectors $= \cos^{-1}\left(\dfrac{2}{\sqrt{5}\sqrt{26}}\right) = 79.9°$ (1 d.p.)



The angle calculated above is angle QPR.