Section 1: Roots and coefficients

Section test

1. The quadratic equation $2z^2+3z-4=0$ has roots α and β.
   Find the values of $\alpha+\beta$ and $\alpha\beta$

2. The quadratic equation $z^2-5z+1=0$ has roots α and β.
   Find the quadratic equation with roots $2\alpha+1$, $2\beta+1$.

3. Find a quadratic equation with roots 0.5 and -2.

4. The cubic equation $3z^3+2z^2-z-3=0$ has roots α, β, γ.
   Find the values of $\sum\alpha$, $\sum\alpha\beta$ and $\alpha\beta\gamma$.

5. Find a cubic equation with roots -2, 1 + 2i, 1 – 2i.

6. The roots of the cubic equation $z^3+z^2-z-1=0$ are α, β, γ. Find a cubic equation with roots 2α, 2β, 2γ.

Questions 7 and 8 are about the cubic equation $3z^3+pz^2+qz+15=0$, which has roots $\alpha, 1-2\alpha, \frac{1}{\alpha}$.

7. Find the value of α.

8. Find the values of p and q.

9. One root of the equation $4z^3-13z+6=0$ is three times another.
   Find the roots of the equation.

10. The roots of the quartic equation $z^4+3z^3-2z+1=0$ are α, β, γ and δ.
    Find the value of $\alpha\beta\gamma+\beta\gamma\delta+\gamma\delta\alpha+\delta\alpha\beta$.

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Solutions to section test

1. For the quadratic equation $2z^2+3z-4=0$, $a = 2$, $b = 3$, $c = -4$.

   The sum of the roots, $\alpha+\beta=-\frac{b}{a}=-\frac{3}{2}$.

   The product of the roots, $\alpha\beta=\frac{c}{a}=\frac{-4}{2}=-2$

2. Let $w=2z+1$, so $z=\frac{w-1}{2}$

   Substituting into $z^2-5z+1=0$:

   $$\left(\frac{w-1}{2}\right)^2-5\left(\frac{w-1}{2}\right)+1=0$$

   $$\frac{(w-1)^2}{4}-\frac{5(w-1)}{2}+1=0$$

   $$(w-1)^2-10(w-1)+4=0$$

   $$w^2-2w+1-10w+10+4=0$$

   $$w^2-12w+15=0$$

3. The sum of the roots is -1.5, so $-\frac{b}{a}=-1.5\Rightarrow b=1.5a$

   The product of the roots is -1, so $\frac{c}{a}=-1\Rightarrow c=-a$

   Let $a = 2$, then $b = 3$ and $c = -2$

   The quadratic equation with these roots is $2z^2+3z-2=0$.

4. For the cubic equation $3z^3+2z^2-z-3=0$, $a = 3$, $b = 2$, $c = -1$, $d = -3$

   $$\sum\alpha=\alpha+\beta+\gamma=-\frac{b}{a}=-\frac{2}{3}$$

   $$\sum\beta\gamma=\beta\gamma+\gamma\alpha+\alpha\beta=\frac{c}{a}=-\frac{1}{3}$$

   $$\alpha\beta\gamma=-\frac{d}{a}=1$$

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5. $\sum\alpha = -2+1+2i+1-2i = 0 \Rightarrow -\frac{b}{a} = 0 \Rightarrow b = 0$

$\sum\beta\gamma = -2(1+2i)-2(1-2i)+(1+2i)(1-2i) = -4+1+4 = 1 \Rightarrow \frac{c}{a} = 1 \Rightarrow c = a$

$\alpha\beta\gamma = -2(1+2i)(1-2i) = -2(1+4) = -10 \Rightarrow -\frac{d}{a} = -10 \Rightarrow d = 10a$

Let $a = 1$, so $b = 0$, $c = 1$ and $d = 10$

The cubic equation is $z^3+z+10=0$

6. Let $w = 2z$, so $z = \frac{w}{2}$

Substituting into $z^3+z^2-z-1=0$:

$$\left(\frac{w}{2}\right)^3+\left(\frac{w}{2}\right)^2-\frac{w}{2}-1=0$$

$$\frac{w^3}{8}+\frac{w^2}{4}-\frac{w}{2}-1=0$$

$$w^3+2w^2-4w-8=0$$

7. For the equation $3z^3+pz^2+qz+15=0$, $a = 3$, $b = p$, $c = q$, $d = 15$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{15}{3} = -5$$

$$\cancel{\alpha}(1-2\alpha)\times\frac{1}{\cancel{\alpha}} = -5$$

$$1-2\alpha = -5$$

$$\alpha = 3$$

8. $\sum\alpha = -\frac{b}{a}$

$3-5+\frac{1}{3} = -\frac{p}{3}$

$p = 5$

$\sum\beta\gamma = \frac{c}{a}$

$-\frac{5}{3}+1-15 = \frac{q}{3}$

$q = -47$

9. For the equation $4z^3-13z+6=0$, $a = 4$, $b = 0$, $c = -13$, $d = 6$

Let the roots be $\alpha, 3\alpha, \beta$

$$\sum\alpha = -\frac{b}{a}$$

$$\alpha+3\alpha+\beta=0$$

$$4\alpha+\beta=0 \qquad ①$$

$\alpha\beta\gamma = -\frac{d}{a}$

$\alpha \times 3\alpha \times \beta = -\frac{6}{4} = -\frac{3}{2}$

$\beta = -\frac{1}{2\alpha^2}$ ②

Substituting ② into ①:

$4\alpha - \frac{1}{2\alpha^2} = 0$

$8\alpha^3 = 1$

$\alpha^3 = \frac{1}{8}$

$\alpha = \frac{1}{2}$

$\beta = -\frac{1}{2\alpha^2} = -\frac{1}{2 \times \frac{1}{4}} = -2$

The roots are $\frac{1}{2}, \frac{3}{2}, -2$

10. $\sum \alpha\beta\gamma = -\frac{d}{a} = 2$