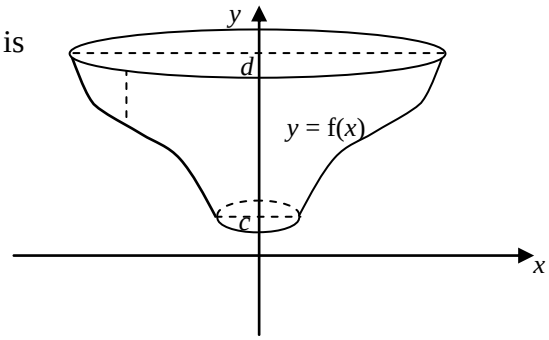
Section 1: Volumes of revolution

Section test

1. The volume of the solid of revolution shown is



(a) $\int_c^d x^2 \, dy$ (b) $\pi\int_c^d y^2 \, dx$

(c) $\int_c^d x \, dy$ (d) $\pi\int_c^d x^2 \, dy$

2. Find the volume of the solid obtained when the line $y = 2x$ between $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. Give your answer in terms of π.

3. Find the volume of the solid obtained when the graph of $y = \frac{2}{x}$ between $y = 1$ and $y = 2$ is rotated through 360° about the y-axis. Give your answer in terms of π.

4. Find the volume of the solid obtained when the graph of $y = x^2 - 1$ between $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. Give your answer in terms of π.

5. The volume of the solid obtained when the graph of $y = 2\sqrt{x}$ between $x = 0$ and $x = a$ is rotated through 360° about the x-axis is 8 cubic units.
The value of a is

(a) $\frac{2}{\pi}$ (b) $\frac{2}{\sqrt{\pi}}$

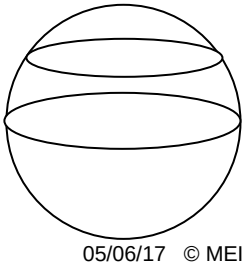
(c) 2 (d) $\frac{1}{\sqrt{\pi}}$

6. The semicircle with centre (2, 0) and radius 2, for which y is positive, is rotated through 360° about the x-axis. The volume of the resulting solid of revolution is given by

(a) $\pi\int_0^2 (4x - x^2)dx$ (b) $\pi\int_0^4 (4x + x^2)dx$

(c) $\pi\int_0^4 (4x - x^2)dx$ (d) $2\pi\int_0^4 (4x - x^2)dx$

7. A sphere of radius 2 cm is cut through by a plane 1 cm from the centre. Find the volume of the 'cap' produced. Give your answer in terms of π.



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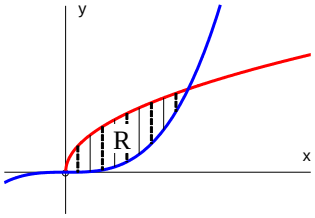
8. The diagram below shows the region R enclosed between the curves $y = \sqrt{x}$ and $y = x^3$.



The region R is rotated through 360° about the *y*-axis. Find the volume of the solid shape formed. Give your answer in terms of π.

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Solutions to section test

1. The solid of revolution is obtained by rotating the area between the graph and the y-axis, between $y = c$ and $y = d$, about the y-axis.

   The volume of this solid is therefore $\pi\int_c^d x^2\,dy$

2. $$\begin{aligned}\text{Volume} &= \int_1^2 \pi y^2 dx \\ &= \pi\int_1^2 (2x)^2 dx \\ &= \pi\int_1^2 4x^2 dx \\ &= \pi\left[\tfrac{4}{3}x^3\right]_1^2 \\ &= \pi\left(\tfrac{32}{3} - \tfrac{4}{3}\right) \\ &= \tfrac{28}{3}\pi\end{aligned}$$

3. $$y = \frac{2}{x} \quad \Rightarrow x = \frac{2}{y}$$

   $$\begin{aligned}\text{Volume} = \int_1^2 \pi x^2 dy &= \pi\int_1^2 \left(\frac{2}{y}\right)^2 dy \\ &= \pi\int_1^2 4y^{-2} dy \\ &= 4\pi\left[-y^{-1}\right]_1^2 \\ &= 4\pi\left(-\tfrac{1}{2} + 1\right) \\ &= 2\pi\end{aligned}$$

4. $$\begin{aligned}\text{Volume} = \int_0^1 \pi y^2 dx &= \pi\int_0^1 \left(x^2 - 1\right)^2 dx \\ &= \pi\int_0^1 \left(x^4 - 2x^2 + 1\right) dx \\ &= \pi\left[\tfrac{1}{5}x^5 - \tfrac{2}{3}x^3 + x\right]_0^1 \\ &= \pi\left(\tfrac{1}{5} - \tfrac{2}{3} + 1\right) \\ &= \tfrac{8}{15}\pi\end{aligned}$$

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5. Volume $= \int_0^a \pi y^2 dx$

$$8 = \pi \int_0^a (2\sqrt{x})^2 dx$$
$$= \pi \int_0^a 4x\, dx$$
$$= \pi \left[2x^2\right]_0^a$$
$$= \pi\left(2a^2 - 0\right)$$
$$= 2\pi a^2$$
$$a^2 = \frac{4}{\pi} \quad \Rightarrow a = \frac{2}{\sqrt{\pi}}$$

6. Equation of circle is $(x-2)^2 + y^2 = 2^2$

$$y^2 = 4 - (x-2)^2 = 4 - (x^2 - 4x + 4) = 4x - x^2$$

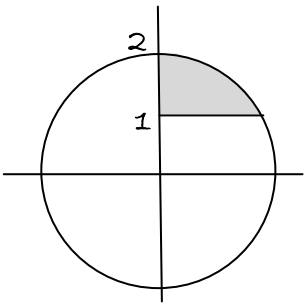
The circle cuts the x-axis at (0, 0) and (4, 0).

Volume of revolution $= \int_0^4 \pi y^2 dx$

$$= \pi \int_0^4 (4x - x^2) dx$$

7. The solid required can be obtained by rotating part of a circle about the y-axis.
The circle has radius 2 and centre (0, 0).
The equation of the circle is $x^2 + y^2 = 2^2$

$$x^2 = 4 - y^2$$



Volume $= \int_1^2 \pi x^2 dy$

$$= \pi \int_1^2 (4 - y^2) dy$$
$$= \pi \left[4y - \tfrac{1}{3}y^3\right]_1^2$$
$$= \pi \left(8 - \tfrac{8}{3} - (4 - \tfrac{1}{3})\right)$$
$$= \pi \left(4 - \tfrac{7}{3}\right)$$
$$= \tfrac{5}{3}\pi$$

8. The curves intersect when $x = 0$ and $x = 1$.

$$y = \sqrt{x} \quad \Rightarrow x = y^2$$

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$y = x^3 \quad \Rightarrow x = y^{\frac{1}{3}}$

$$\begin{aligned}
\text{Volume} &= \int_0^1 \pi x^2 dy \\
&= \pi \int_0^1 \left((y^{\frac{1}{3}})^2 - (y^2)^2 \right) dy \\
&= \pi \int_0^1 \left(y^{\frac{2}{3}} - y^4 \right) dy \\
&= \pi \left[\tfrac{3}{5} y^{\frac{5}{3}} - \tfrac{1}{5} y^5 \right]_0^1 \\
&= \pi \left(\tfrac{3}{5} - \tfrac{1}{5} - 0 \right) \\
&= \tfrac{2}{5} \pi
\end{aligned}$$