Section 1: Summing series

Section test

In Question 1-3, S_1 denotes $\sum_{r=1}^{n} r$, S_2 denotes $\sum_{r=1}^{n} r^2$ and S_3 denotes $\sum_{r=1}^{n} r^3$.

1. The sum $\sum_{r=1}^{n}\left(4r^2(r+1)+2\right)$ is equal to

(a) $4S_3 + 4S_2 + 2$
(b) $4S_3 + 4S_2 + 2n$
(c) $4S_3 + S_2 + 2$
(d) $4S_2(S_1 + 1) + 2n$

2. The sum $\sum_{r=1}^{n}\left((r+1)(r-1)+1\right)$ is equal to

(a) $(S_1 + 1)(S_1 - 1) + 1$
(b) S_2
(c) $(S_1 + 1)(S_1 - 1) + n$
(d) $S_1^2 - n^2 + n$

3. The sum $\sum_{r=1}^{n}(3r-1)(2r+1)$ is equal to

(a) $6S_2 + S_1 - 1$
(b) $6S_2 - S_1 - n$
(c) $6S_2 + S_1 - n$
(d) $(3S_1 - 1)(2S_1 + 1)$

4. Find $1+2+3+...+500$.

5. Find $\sum_{r=1}^{200} r^3$.

6. Find $\sum_{r=1}^{100}(r^2+1)$

7. Find $\sum_{r=1}^{50}(r^3-r^2+r-2)$

8. Find the sum $\sum_{r=1}^{n}(2r+1)(2r-1)$.

9. Find the sum $\sum_{r=1}^{n} r(r+2)(r-2)$.

10. Find the sum $(1\times3)+(2\times5)+(3\times7)+...+n(2n+1)$.

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Solutions to section test

1. $$\sum_{r=1}^{n}\left(4r^2(r+1)+2\right)=\sum_{r=1}^{n}(4r^3+4r^2+2)$$
$$=4\sum_{r=1}^{n}r^3+4\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}2$$
$$=4S_3+4S_2+2n$$

2. $$\sum_{r=1}^{n}((r+1)(r-1)+1)=\sum_{r=1}^{n}\left(r^2-1+1\right)$$
$$=\sum_{r=1}^{n}r^2$$
$$=S_2$$

3. $$\sum_{r=1}^{n}(3r-1)(2r+1)=\sum_{r=1}^{n}(6r^2+r-1)$$
$$=6\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r-\sum_{r=1}^{n}1$$
$$=6S_2+S_1-n$$

4. $$\sum_{r=1}^{n}r=\tfrac{1}{2}n(n+1)$$
$$\sum_{r=1}^{500}r=\tfrac{1}{2}\times 500\times 501$$
$$=125250$$

5. $$\sum_{r=1}^{n}r^3=\tfrac{1}{4}n^2(n+1)^2$$
$$\sum_{r=1}^{200}r^3=\tfrac{1}{4}\times 200^2\times 201^2$$
$$=404010000$$

6. $$\sum_{r=1}^{n}r^2=\tfrac{1}{6}n(n+1)(2n+1)$$
$$\sum_{r=1}^{100}(r^2+1)=\sum_{r=1}^{100}r^2+\sum_{r=1}^{100}1$$
$$=\tfrac{1}{6}\times 100\times 101\times 201+100$$
$$=338450$$

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7. $$\sum_{r=1}^{50}(r^3-r^2+r-2)=\sum_{r=1}^{50}r^3-\sum_{r=1}^{50}r^2+\sum_{r=1}^{50}r-\sum_{r=1}^{50}2$$
$$=\tfrac{1}{4}\times 50^2\times 51^2-\tfrac{1}{6}\times 50\times 51\times 101+\tfrac{1}{2}\times 50\times 51-2\times 50$$
$$=1583875$$

8. $$\sum_{r=1}^{n}(2r+1)(2r-1)=\sum_{r=1}^{n}(4r^2-1)=4\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}1$$
$$=4\times\tfrac{1}{6}n(n+1)(2n+1)-n$$
$$=\tfrac{2}{3}n(n+1)(2n+1)-n$$
$$=\tfrac{1}{3}n\left(4n^2+6n+2-3\right)$$
$$=\tfrac{1}{3}n\left(4n^2+6n-1\right)$$
$$=\tfrac{4}{3}n^3+2n^2-\tfrac{1}{3}n$$

9. $$\sum_{r=1}^{n}r(r+2)(r-2)=\sum_{r=1}^{n}(r^3-4r)=\sum_{r=1}^{n}r^3-4\sum_{r=1}^{n}r$$
$$=\tfrac{1}{4}n^2(n+1)^2-4\times\tfrac{1}{2}n(n+1)$$
$$=\tfrac{1}{4}n(n+1)\left(n(n+1)-8\right)$$
$$=\tfrac{1}{4}n(n+1)(n^2+n-8)$$

10. $$(1\times 3)+(2\times 5)+(3\times 7)+\ldots+n(2n+1)=\sum_{r=1}^{n}r(2r+1)$$
$$=\sum_{r=1}^{n}(2r^2+r)$$
$$=2\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r$$
$$=2\times\tfrac{1}{6}n(n+1)(2n+1)+\tfrac{1}{2}n(n+1)$$
$$=\tfrac{1}{6}n(n+1)\left(4n+2+3\right)$$
$$=\tfrac{1}{6}n(n+1)\left(4n+5\right)$$
$$=\tfrac{2}{3}n^3+\tfrac{3}{2}n^2+\tfrac{5}{6}n$$