Section 3: Matrices and simultaneous equations

Section test

1. Which of the following sets of equations are consistent?

   (i) $3x - 2y = 6$, $2x + y = 4$ (ii) $-3x + 2y = -1$, $9x - 6y = -3$ (iii) $3x - 2y = 1$, $2x - y = 1$

2. The equations

   $$3x - y = 4$$
   $$2y - 6x = 1$$

   have

   (a) no solutions (b) a unique solution
   (c) infinitely many solutions (d) two solutions

3. A point P in three dimensional space is mapped to the point (-3, 2, 3) by the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 3 & -2 & -1 \\ 1 & 1 & 0 \end{pmatrix}$. Find the coordinates of P.

Questions 4 and 5 are about the matrices

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 0 & 1 \\ 1 & 2 & k \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -2 & 8-k & 1 \\ 1-2k & 3k-4 & 5 \\ 4 & -5 & -2 \end{pmatrix}$$

4. By calculating the matrix **AB**, find the value of k for which the matrix **A** has no inverse matrix.

5. In the case for which $k = 5$, find the solution of the matrix equation

   $$\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

6. Match the sets of simultaneous equations with the geometrical arrangements of three planes.

   A
   $$2x + 3y - z = 1$$
   $$x - 2y + 3z = 2$$
   $$-2x - 3y + z = 4$$

   B
   $$x + 2y + 3z = 1$$
   $$4x + 3y + 2z = 0$$
   $$x + 3y + 3y = 2$$

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C $x+2y+3z=2$
$x+4y-z=1$
$x+3y+z=3$

D $x-y-3z=0$
$-3x+3y+9z=2$
$2x-2y-6z=5$

E $5x+4y+z=-4$
$3x+y+2z=6$
$x-y+2z=10$

(i) Planes meet at a single point
(ii) Sheaf of planes
(iii) Triangular prism
(iv) Three parallel planes
(v) Two parallel planes with the third intersecting them

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Solutions to section test

1. (i) The equations can be written as the matrix equation

$$\begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}.$$

$\det\begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix} = 7$, so the equations have a unique solution, and therefore they are consistent.

(ii) The equations can be written as the matrix equation

$$\begin{pmatrix} -3 & 2 \\ 9 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

$\det\begin{pmatrix} -3 & 2 \\ 9 & -6 \end{pmatrix} = 0$, so the equations do not have a unique solution.

The second equation can be divided by -3 to give $-3x + 2y = 1$. This contradicts the first equation, so the equations are inconsistent.

(iii) The equations can be written as the matrix equation

$$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\det\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \neq 0$, so the equations have a unique solution, and so are consistent.

2. The equations can be written as the matrix equation $\begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\det\begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} = 0$, so the equations do not have a unique solution.

The first equation gives $y = 3x - 4$

The second equation gives $y = 3x + \frac{1}{2}$

so the equations are inconsistent, and therefore have no solution.

3. $$\begin{pmatrix} 1 & 1 & 2 \\ 3 & -2 & -1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$$

Inverse matrix is $\begin{pmatrix} 0.1 & 0.2 & 0.3 \\ -0.1 & -0.2 & 0.7 \\ 0.5 & 0 & -0.5 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ -0.1 & -0.2 & 0.7 \\ 0.5 & 0 & -0.5 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

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P is the point (1, 2, -3)

4. $$\begin{pmatrix} 3 & 1 & 4 \\ 2 & 0 & 1 \\ 1 & 2 & k \end{pmatrix}\begin{pmatrix} -2 & 8-k & 1 \\ 1-2k & 3k-4 & 5 \\ 4 & -5 & -2 \end{pmatrix} = \begin{pmatrix} 11-2k & 0 & 0 \\ 0 & 11-2k & 0 \\ 0 & 0 & 11-2k \end{pmatrix}$$

The inverse of A is therefore $\frac{1}{11-2k}\mathbf{B}$

A has no inverse in the case for which $11-2k=0$

$$k = 5.5$$

5. In the case for which $k = 5$, $\mathbf{A}^{-1} = \mathbf{B} = \begin{pmatrix} -2 & 3 & 1 \\ -9 & 11 & 5 \\ 4 & -5 & -2 \end{pmatrix}$

$$\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 & 3 & 1 \\ -9 & 11 & 5 \\ 4 & -5 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -19 \\ 8 \end{pmatrix}$$

The solution of the equation is $x = -4$, $y = -19$, $z = 8$.

6. A (v)
   B (i)
   C (iii)
   D (iv)
   E (ii)