Section 2: Matrices and transformations

Section test

1. The matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ represents the transformation

(a) Reflection in the line $y = -x$
(b) Rotation through 180° about the origin
(c) Reflection in the line $y = x$
(d) Rotation through 90° clockwise about the origin

2. The matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents the transformation

(a) Reflection in the x axis
(b) Reflection in the y axis
(c) Rotation through 90° anticlockwise about the origin
(d) Rotation through 90° clockwise about the origin

3. A two-way stretch, scale factor 4 in the x direction and scale factor 2 in the y direction is represented by the matrix

(a) $\begin{pmatrix} 0 & 4 \\ 2 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$
(c) $\begin{pmatrix} 0 & 2 \\ 4 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$

4. A reflection in the x axis is represented by the matrix

(a) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

5. Which of these matrices represent rotations?

$\mathbf{A} = \begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -0.8 & 0.6 \\ -0.6 & -0.8 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix}$

$\mathbf{D} = \begin{pmatrix} -0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix}$ $\mathbf{E} = \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$

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6. A triangle has vertices A (2, 0), B (3, 1) and C (-1, 3)
   The triangle is transformed using the matrix $\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$.
   What are the vertices of the image?

7. In a transformation, the point (1, 0) is mapped to (1, 4) and the point (0, 1) is mapped to itself.
   What is the image of the point (-2, 3)?

8. The matrix $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ represents which of the following transformations?
   (a) reflection in $x = 0$
   (b) reflection in $y = 0$
   (c) reflection in $z = 0$
   (d) rotation through 90° clockwise about the x-axis
   (e) rotation through 90° clockwise about the y-axis
   (f) rotation through 90° clockwise about the z-axis

In Questions 9 – 10, the transformation A is represented by the matrix $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$
and the transformation B is represented by the matrix $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}$

9. The composite transformation "A followed by B" is represented by the matrix

(a) $\begin{pmatrix} 2 & -4 \\ 5 & -7 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 4 \\ -7 & -11 \end{pmatrix}$
(c) $\begin{pmatrix} 0 & 6 \\ -1 & -9 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 7 \\ -6 & -9 \end{pmatrix}$

10. The composite transformation "***B*** followed by ***A***" is represented by the matrix

(a) $\begin{pmatrix} 2 & -4 \\ 5 & -7 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 4 \\ -7 & -11 \end{pmatrix}$
(c) $\begin{pmatrix} 0 & 6 \\ -1 & -9 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 7 \\ -6 & -9 \end{pmatrix}$

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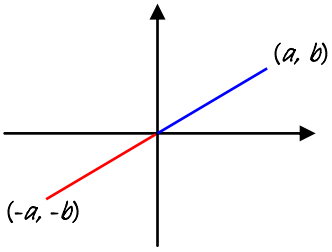
Solutions to section test

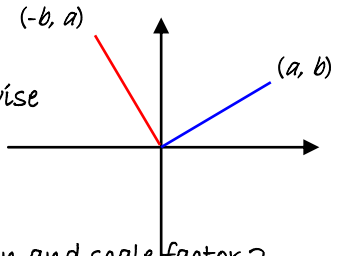
1. $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -a \\ -b \end{pmatrix}$ so the image of (a, b) is $(-a, -b)$.

   The matrix represents a rotation of 180° about the origin.



2. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -b \\ a \end{pmatrix}$ so the image of (a, b) is $(-b, a)$

   The matrix represents a rotation through 90° anticlockwise about the origin.



3. Under a two-way stretch, scale factor 4 in the x direction and scale factor 2 in the y direction, the point $(1, 0)$ is mapped to $(4, 0)$ and the point $(0, 1)$ is mapped to $(0, 2)$.

   So the matrix representing this transformation is $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$.

4. Under a reflection in the x-axis, the point $(1, 0)$ is mapped to itself, and the point $(0, 1)$ is mapped to $(0, -1)$.

   So the matrix representing this transformation is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

5. The general rotation matrix is $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

   So the elements on the leading diagonal must be the same, with the same sign. This is the case in matrices A, B, D and E.
   The elements on the other diagonal must be the same but have opposite signs. This is the case in matrices A, B and E.
   So the matrices which represent rotations are A, B and E.

6. $\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 3 & -1 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 7 & 1 \\ 6 & 13 & 9 \end{pmatrix}$

   The vertices of the image are A′ (4, 6), B′ (7, 13) and C′ (1, 9).

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7. The matrix for the transformation is $\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

so the image of (-2, 3) is (-2, -5)

8. Rotation about the y-axis through 90° clockwise.

9. The composite transformation "A followed by B" is represented by the matrix

$$BA = \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ -1 & -9 \end{pmatrix}$$

10. The composite transformation "B followed by A" is represented by the matrix

$$AB = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ -7 & -11 \end{pmatrix}$$

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