Section 1: Determinants and inverses

Section test

1. Find the determinant of the matrix $\begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$.

2. Which of the following matrices are singular?

   (i) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (v) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3. Which of the following matrices are singular?

   (i) $\begin{pmatrix} 2 & 3 & -1 \\ 3 & 1 & 2 \\ 1 & -1 & 2 \end{pmatrix}$ (ii) $\begin{pmatrix} 2 & 3 & -1 \\ 3 & 1 & 2 \\ 1 & -1 & 3 \end{pmatrix}$

4. A triangle has vertices (1, 2), (3, 2) and (2, 6).

   The triangle is transformed by the matrix $\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$.

   Find the area of the image of the triangle.

5. A cube with volume 8 cubic units is transformed in 3D space using the matrix

   $\begin{pmatrix} 1 & 4 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 2 \end{pmatrix}$

   Find the volume of the image.

In the following questions, **A** is the matrix $\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ and **B** is the matrix $\begin{pmatrix} 1 & 1 \\ 4 & 2 \end{pmatrix}$.

6. $\mathbf{A}^{-1}$ =

(a) $\begin{pmatrix} -3 & 4 \\ 1 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$

(c) $\frac{1}{2}\begin{pmatrix} -3 & 4 \\ 1 & -2 \end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$

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7. $\mathbf{B}^{-1}$ =

(a) $\frac{1}{2}\begin{pmatrix} 1 & -4 \\ -1 & 2 \end{pmatrix}$ (b) $\frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 1 \end{pmatrix}$

(c) $\frac{1}{2}\begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix} -2 & 1 \\ 4 & -1 \end{pmatrix}$

8. $(\mathbf{AB})^{-1}$ =

(a) $\frac{1}{4}\begin{pmatrix} -8 & 5 \\ 12 & -7 \end{pmatrix}$ (b) $\frac{1}{4}\begin{pmatrix} 8 & -3 \\ -20 & 7 \end{pmatrix}$

(c) $\frac{1}{4}\begin{pmatrix} -8 & 3 \\ 20 & -7 \end{pmatrix}$ (d) $\frac{1}{4}\begin{pmatrix} 8 & -5 \\ -12 & 7 \end{pmatrix}$

9. $(\mathbf{BA})^{-1}$ =

(a) $\frac{1}{4}\begin{pmatrix} 8 & -5 \\ -12 & 7 \end{pmatrix}$ (b) $\frac{1}{4}\begin{pmatrix} 8 & -3 \\ -20 & 7 \end{pmatrix}$

(c) $\frac{1}{4}\begin{pmatrix} -8 & 3 \\ 20 & -7 \end{pmatrix}$ (d) $\frac{1}{4}\begin{pmatrix} -8 & 5 \\ 12 & -7 \end{pmatrix}$

10. The plane is transformed by the matrix $\mathbf{M} = \begin{pmatrix} -3 & 6 \\ 1 & -2 \end{pmatrix}$. The whole plane is mapped to the line

(a) $-3x + y = 0$ (b) $3x + y = 0$

(c) $x - 3y = 0$ (d) $x + 3y = 0$

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Solutions to section test

1. $\det\begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix} = (2\times -5) - (-3\times 4) = -10 + 12 = 2$

2. (i) $\det\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = (1\times 0) - (1\times 0) = 0 - 0 = 0$

   (ii) $\det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (1\times 1) - (0\times 0) = 1 - 0 = 1$

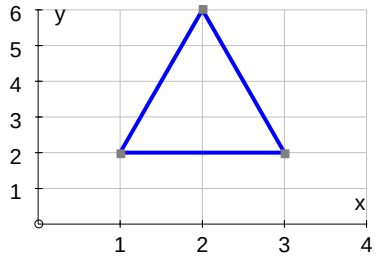
   (iii) $\det\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = (0\times 1) - (0\times 1) = 0 - 0 = 0$

   (iv) $\det\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = (1\times 1) - (1\times 1) = 1 - 1 = 0$

   (v) $\det\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (0\times 0) - (1\times 1) = 0 - 1 = -1$

   Matrices (i), (iii) and (iv) are singular since their determinants are zero.

3. The determinant of (i) is zero, and the determinant of (ii) is not. So (i) is singular and (ii) is not.

4. 

   Area of triangle $= \frac{1}{2}\times 2\times 4 = 4$
   Determinant of matrix $= -1 - 6 = -7$
   Area scale factor of transformation $= 7$
   Area of image $= 4\times 7 = 28$

5. The determinant of the matrix is 25, so the volume of the image is $8 \times 25 = 200$ cubic units.

6. $\det A = \det\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} = (3\times 2) - (1\times 4) = 2$

   $A^{-1} = \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$.

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7. $\det \mathbf{B} = \det\begin{pmatrix} 1 & 1 \\ 4 & 2 \end{pmatrix} = (1\times 2) - (1\times 4) = -2$

$$\mathbf{B}^{-1} = -\frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2 & 1 \\ 4 & -1 \end{pmatrix}$$

8. $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

$$= \frac{1}{2}\begin{pmatrix} -2 & 1 \\ 4 & -1 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} -8 & 5 \\ 12 & -7 \end{pmatrix}$$

9. $(\mathbf{BA})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$

$$= \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}\frac{1}{2}\begin{pmatrix} -2 & 1 \\ 4 & -1 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} -8 & 3 \\ 20 & -7 \end{pmatrix}$$

10. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -3 & 6 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3x+6y \\ x-2y \end{pmatrix}$

$x' = -3(x-2y) = -3y'$

The whole plane is mapped to the line $x = -3y$, i.e. $x + 3y = 0$.