Section 2: Loci in the complex plane

Section test

Questions 1 – 3 are about the following loci:

P: $|z-3-4\text{i}|=2$ Q: $|z-3-4\text{i}|=|z|$

R: $\arg(z-3-4\text{i})=\dfrac{\pi}{3}$

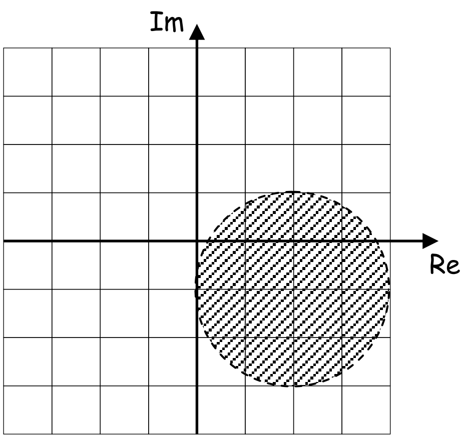
1. Which of the loci above represent a circle?

2. Which of the loci above represent a straight line (or part of a straight line)?

3. Which of the loci above are satisfied by the point 3 + 4i?

4. Describe the set of points for which $|z-2+3\text{i}|=4$.

5.

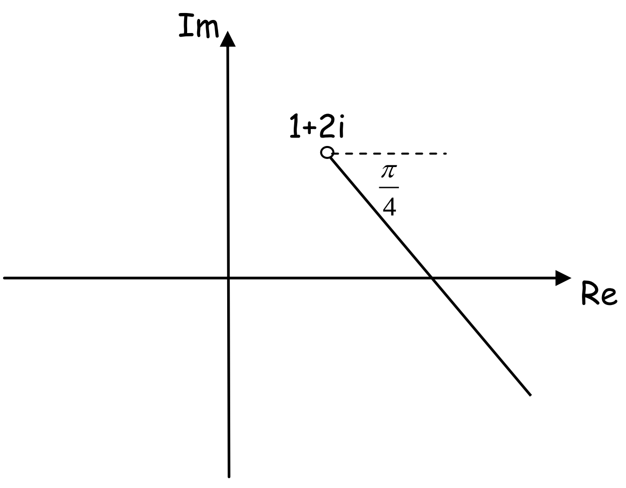


The shaded area in the Argand diagram represents the points z for which

(a) $|z-2+\text{i}|\leq 2$ (b) $|z-2+\text{i}|<2$

(c) $|z+2-\text{i}|<2$ (d) $|z+2-\text{i}|\leq 2$

6.



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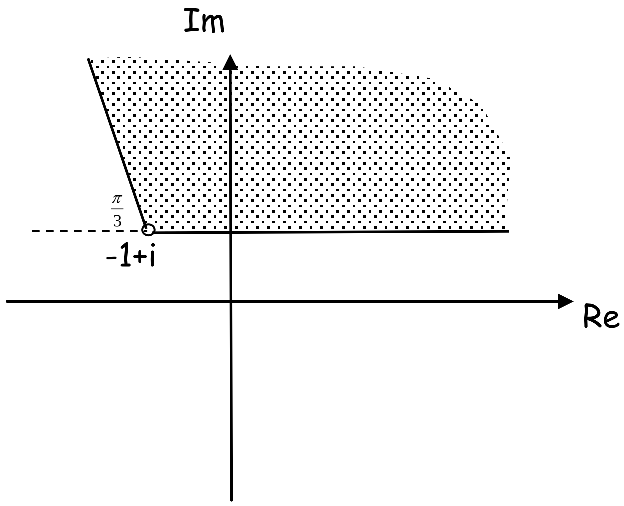
The bold half-line in the diagram above shows the set of points z for which

(a) $\arg(z-1-2\mathrm{i}) = -\frac{\pi}{4}$ (b) $\arg(z-1-2\mathrm{i}) = \frac{\pi}{4}$

(c) $\arg(z+1+2\mathrm{i}) = -\frac{\pi}{4}$ (d) $\arg(z+1+2\mathrm{i}) = \frac{\pi}{4}$

7.



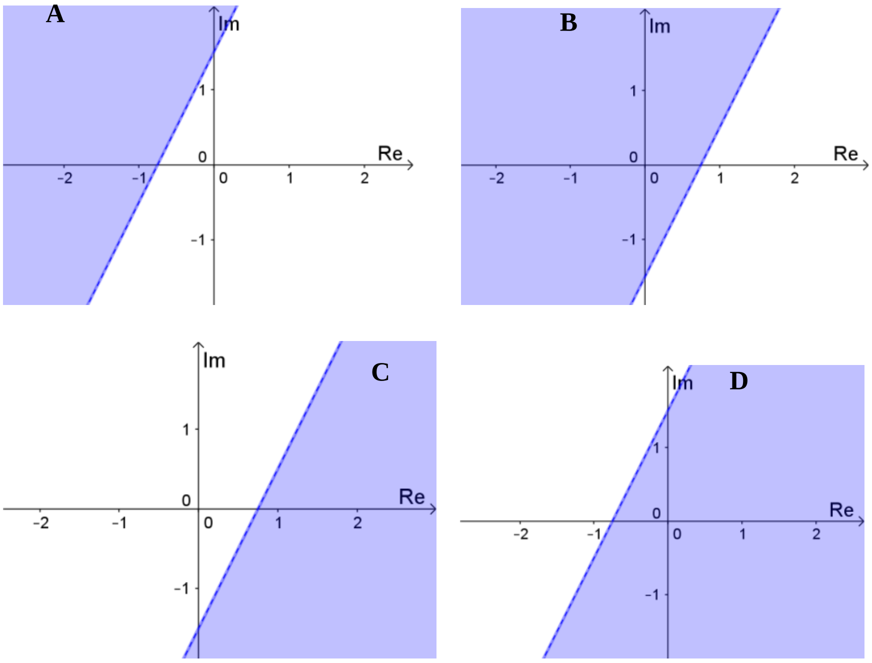
The shaded area in the diagram above shows the set of points z for which

(a) $\arg(z+1-\mathrm{i}) \le \frac{2\pi}{3}$ (b) $\arg(z-1+\mathrm{i}) \le \frac{2\pi}{3}$

(c) $0 \le \arg(z-1+\mathrm{i}) \le \frac{2\pi}{3}$ (d) $0 \le \arg(z+1-\mathrm{i}) \le \frac{2\pi}{3}$

8. Match the diagrams below with the following loci:

(i) $|z-2| < |z-\mathrm{i}|$ (ii) $|z-2| > |z-\mathrm{i}|$ (iii) $|z+2| < |z+\mathrm{i}|$ (iv) $|z+2| > |z+\mathrm{i}|$



9. The sketch, on an Argand diagram, of the locus of points satisfying $|z-3| = |z+1-4\text{i}|$ is

(a) the line joining the points (3, 0) and (-1, 4)
(b) the line joining the points (3, 0) and (1, -4)
(c) the line joining the points (-1, 0) and (1, 2)
(d) the line joining the points (-3, 0) and (1, -4)

10. Find the complex number which satisfies both $|z| = 2$ and $|z-\text{i}| = |z-3\text{i}|$.

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Solutions to section test

1. For P, z represents all points whose distance from the point 3 + 4i is 2. This means that P is a circle.

   For Q, z represents all points whose distance from the point 3 + 4i is equal to its distance from the origin. This means that Q is the perpendicular bisector of a line joining the origin and the point 3 + 4i.

   For R, z represents all points for which a line from the point to 3 + 4i makes an angle of $\frac{\pi}{3}$ with the real axis. This is the half-line from the point 3 + 4i in the direction making an angle of $\frac{\pi}{3}$ with the real axis.

   Therefore P represents a circle.

2. From the solution to question 1, Q is a straight line since it is a perpendicular bisector, and R is a half- line so is part of a straight line.

3. The point 3 + 4i is the centre of the circle represented by P, so it does not lie on the circle itself.
   For Q, for the point 3 + 4i the value of $|z-3-4i|$ is zero, but the value of $|z|$ is 5, so 3 + 4i cannot lie on this locus.
   R represents a line starting from 3 + 4i, but the point 3 + 4i is not part of the locus since the argument of zero is not defined.
   Therefore the point does not lie on any of the loci.

4. $|z-2+3i|=4$

   $|z-(2-3i)|=4$

   This means that the distance of the point z from the point 2 – 3i is always 4.
   Therefore the set of points is a circle, centre 2 – 3i, radius 4.

5. The shaded area is the area inside the circle with centre (2, -1) and radius 2.
   So this is the locus of a point z whose distance from the point 2 – i is always less than 2.
   This is the set of points given by $|z-(2-i)|<2$

   $$|z-2+i|<2$$

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6. The line starts from the point 1 + 2i and goes in the direction $-\frac{\pi}{4}$.

   The set of points is therefore $\arg(z-(1+2i)) = -\frac{\pi}{4}$

   $$\arg(z-1-2i) = -\frac{\pi}{4}$$

7. The lines bounding the region both start from the point -1 + i and have directions 0 and $\frac{2\pi}{3}$ respectively.

   These lines are given by $\arg(z+1-i)=0$ and $\arg(z+1-i)=\frac{2\pi}{3}$.

   So the shaded region is given by

   $$0 \le \arg(z+1-i) \le \frac{2\pi}{3}$$

8. (i) This locus represents the points nearer to (2, 0) than to (0, 1).
   This is diagram C

   (ii) This locus represents the points nearer to (0, 1) than to (2, 0).
   This is diagram B

   (iii) This locus represents the points nearer to (-2, 0) than to (0, -1).
   This is diagram A

   (iv) This locus represents the points nearer to (0, -1) than to (-2, 0).
   This is diagram D

9. $|z-3| = |z+1-4i|$

   $|z-3| = |z-(-1+4i)|$

   This means that the distance of the point representing z from the point representing 3 (the point (3, 0)) is equal to the distance of the point representing z from the point representing -1 + 4i (the point (-1, 4)). The locus of z is therefore the perpendicular bisector of the points (3, 0) and (-1, 4).
   The midpoint of (3, 0) and (-1, 4) is (1, 2), so the locus passes through (1, 0).

   The gradient of the line joining (3, 0) and (-1, 4) is $\frac{4}{-4} = -1$, so the gradient of the perpendicular bisector is 1.
   The equation of the perpendicular bisector is therefore $y-2 = x-1$

   $$y = x+1$$

   The locus is therefore the line joining any two points satisfying $y = x+1$, such as (-1, 0) and (1, 2).

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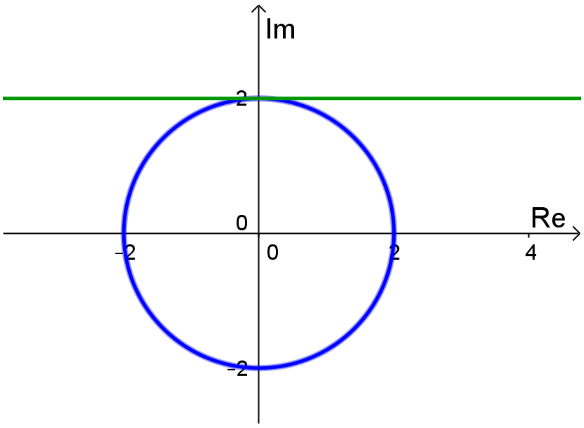
10. The locus $|z| = 2$ represents a circle centre the origin, radius 2, and the locus $|z - i| = |z - 3i|$ represents the perpendicular bisector of the points i and $3i$.



From the diagram it can be seen that the point which satisfies both equations is (0, 2), i.e. the complex number $2i$.