Section 1: Modulus and argument

Section test

1. The modulus of the complex number $z = 2 - 5\mathrm{i}$ is

(a) $\sqrt{29}$ (b) 29

(c) 7 (d) $\sqrt{7}$

2. The principal argument of the complex number $-1 + \sqrt{3}\mathrm{i}$ is

(a) $\frac{2\pi}{3}$ (b) $-\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{3}$

3. The principal argument of the complex number $2 - 2\mathrm{i}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $-\frac{\pi}{4}$ (d) $-\frac{3\pi}{4}$

4. Find the complex number with modulus 2 and argument -1.5.

5. The modulus and principal argument of the complex number $z = -2(\cos\alpha - \mathrm{i}\sin\alpha)$ (where $0 < \alpha \le \frac{\pi}{2}$) are, respectively,

(a) 2, $-\alpha$ (b) 2, $\pi - \alpha$

(c) -2, α (d) 2, $\alpha + \pi$

Questions 6 and 7 are about the complex numbers $z = 5\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)$ and $w = \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}$.

6. The complex number wz is given by

(a) $6\left(\cos\frac{\pi^2}{9} + \mathrm{i}\sin\frac{\pi^2}{9}\right)$ (b) $6\left(\cos\frac{5\pi}{6} + \mathrm{i}\sin\frac{5\pi}{6}\right)$

(c) $5\left(\cos\frac{\pi^2}{9} + \mathrm{i}\sin\frac{\pi^2}{9}\right)$ (d) $5\left(\cos\frac{5\pi}{6} + \mathrm{i}\sin\frac{5\pi}{6}\right)$

7. The complex number $\frac{z}{w}$ is given by

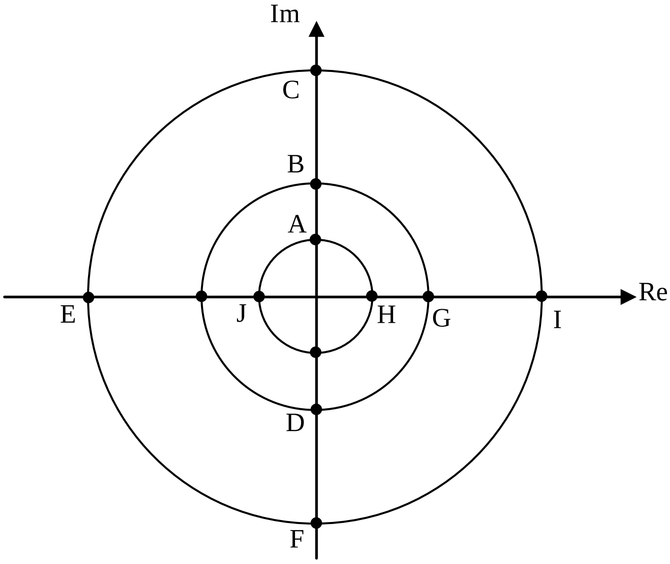
(a) $5\left(\cos\frac{5\pi}{6}+\mathrm{i}\sin\frac{5\pi}{6}\right)$

(b) $4\left(\cos\frac{\pi}{2}+\mathrm{i}\sin\frac{\pi}{2}\right)$

(c) $5\left(\cos\frac{\pi}{2}+\mathrm{i}\sin\frac{\pi}{2}\right)$

(d) $4\left(\cos\frac{5\pi}{6}+\mathrm{i}\sin\frac{5\pi}{6}\right)$

Questions 8, 9 and 10 are about the Argand diagram below which shows three concentric circles, all centred at the origin, with radii 2, 4 and 8.



8. Which point represents the result of cubing the complex number at A?

9. Which point represents the result of multiplying the complex number at A by the complex number at B?

10. Which point represents the result of dividing the complex number at F by the complex number at D?

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Solutions to section test

1. $|2-5i| = \sqrt{2^2+5^2} = \sqrt{29}$

2. $z = -1+\sqrt{3}i$ is in the second quadrant.

   Principal argument $= \arctan\left(\frac{\sqrt{3}}{-1}\right)+\pi$

   $= -\frac{\pi}{3}+\pi$

   $= \frac{2\pi}{3}$

3. $z = 2-2i$ is in the fourth quadrant.

   Principal argument $= \arctan\left(\frac{-2}{2}\right) = \arctan(-1) = -\frac{\pi}{4}$

4. Let $z = x+iy$

   $r = 2, \quad \theta = -1.5$

   $x = r\cos\theta = 2\cos(-1.5) = 0.14$ (2 d.p.)

   $y = r\sin\theta = 2\sin(-1.5) = -1.99$ (2 d.p.)

   The complex number with modulus 2 and argument -1.5 is 0.14 – 1.99i

5. $z = -2(\cos\alpha - i\sin\alpha)$

   $= 2(-\cos\alpha + i\sin\alpha)$

   $= 2(\cos(\pi-\alpha)+i\sin(\pi-\alpha))$

   The modulus of z is 2 and the argument is $\pi-\alpha$.

6. $|z| = 5$ and $\arg z = \frac{2\pi}{3}$

   $|w| = 1$ and $\arg w = \frac{\pi}{6}$

   $|wz| = |w||z| = 1\times 5 = 5$

   $\arg(wz) = \arg w + \arg z = \frac{2\pi}{3}+\frac{\pi}{6} = \frac{5\pi}{6}$

   $wz = 5\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

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7. $\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = \frac{5}{1} = 5$

$\arg\frac{z}{w} = \arg z - \arg w = \frac{2\pi}{3} - \frac{\pi}{6} = \frac{\pi}{2}$

$\frac{z}{w} = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

8. The complex number at A has modulus 2 and argument $\frac{\pi}{2}$. The cube of the complex number at A has modulus $2^3 = 8$ and argument $3 \times \frac{\pi}{2} = \frac{3\pi}{2}$
So the cube of the complex number at A is the complex number at F.

9. The complex number at A has modulus 2 and argument $\frac{\pi}{2}$, and the complex number at B has modulus 4 and argument $\frac{\pi}{2}$. When the complex number at A is multiplied by the complex number at B, the length of vector OA is multiplied by 4 and it is rotated through $\frac{\pi}{2}$ in an anticlockwise direction, resulting in the vector OE. So the result of multiplying the complex number at A by the complex number at B is the complex number at E.

10. The complex number at F has modulus 8 and argument $-\frac{\pi}{2}$. The complex number at D has modulus 4 and argument $-\frac{\pi}{2}$. When the complex number at F is divided by the complex number at D, the length of vector OF is divided by 4 and is rotated through an angle of $-\frac{\pi}{2}$ clockwise, i.e. $\frac{\pi}{2}$ anticlockwise, resulting in the vector OH. So the result of dividing the complex number at F by the complex number at D is the complex number at H.