Section 1: Introduction to complex numbers

Section test

1. Simplify $(3+4\text{i})-(2-\text{i})$.

2. Simplify $\left[(1-2\text{i})+(1+\text{i})\right](-3+\text{i})$

3. The roots of the equation
$$z^2+6z+10=0$$
are

(a) 3 + i, 3 – i
(b) 3 + 2i, 3 – 2i
(c) -3 + 2i, -3 – 2i
(d) -3 + i, -3 – i

4. Given that $p+q\text{i}=\dfrac{1}{12-5\text{i}}$, find the values of p and q.

5. Which of the following complex numbers is not equal to the others?

(a) 2 – 3i
(b) $\dfrac{13}{2+3\text{i}}$
(c) $\dfrac{13}{2-3\text{i}}$
(d) $\dfrac{3+2\text{i}}{\text{i}}$

6. Given that $z=\dfrac{3+4\text{i}}{2-3\text{i}}$, find the complex number w which satisfies the equation $zw = 1$.

7. Solve the equation
$$(3-\text{i})(z+4-2\text{i})=10+20\text{i}$$

8. Which of the following statements are true?

(a) $\text{i}^4=1$
(b) $\dfrac{1}{\text{i}^3}-\text{i}=0$
(c) $\dfrac{1}{\text{i}}+\text{i}^3=0$
(d) $\dfrac{1}{\text{i}^2}=\text{i}^2$

9. Find the values of a and b (with $a > 0$) which satisfy
$$(a+b\text{i})^2=5+12\text{i}$$

10. Which of the following statements are true for all $z\neq 0$?

(a) $z + z^*$ is real
(b) $z - z^*$ is real
(c) zz^* is real
(d) $\dfrac{z}{z^*}$ is real

Solutions to section test

1. $$\begin{aligned} 3+4i-(2-i) &= 3+4i-2+i \\ &= (3-2)+(4i+i) \\ &= 1+5i \end{aligned}$$

2. $$\begin{aligned} [(1-2i)+(1+i)](-3+i) &= (2-i)(-3+i) \\ &= -6+3i+2i-i^2 \\ &= -6+5i+1 \\ &= -5+5i \end{aligned}$$

3. $$\begin{aligned} z &= \frac{-6\pm\sqrt{36-4\times1\times10}}{2} \\ &= \frac{-6\pm\sqrt{-4}}{2} \\ &= \frac{-6\pm2i}{2} \\ &= -3\pm i \end{aligned}$$

4. $$\begin{aligned} p+qi &= \frac{1}{12-5i} \\ &= \frac{12+5i}{(12-5i)(12+5i)} \\ &= \frac{12+5i}{144+25} = \frac{12}{169}+\frac{5}{169}i \end{aligned}$$

   So $p=\frac{12}{169}$, $q=\frac{5}{169}$.

5. $$\frac{13}{2-3i} = \frac{13(2+3i)}{(2-3i)(2+3i)} = \frac{26+39i}{13} = 2+3i$$
   $$\frac{13}{2+3i} = \frac{13(2-3i)}{(2+3i)(2-3i)} = \frac{26-39i}{13} = 2-3i$$
   $$\frac{3+2i}{i} = \frac{(3+2i)i}{-1} = \frac{3i-2}{-1} = 2-3i$$

   So $\frac{13}{2-3i}$ is not equal to the others.

6. $w = \dfrac{2-3i}{3+4i}$

$= \dfrac{(2-3i)(3-4i)}{(3+4i)(3-4i)}$

$= \dfrac{6-17i-12}{25}$

$= \dfrac{-6-17i}{25}$

7. $(3-i)(z+4-2i) = 10+20i$

$z+4-2i = \dfrac{10+20i}{3-i}$

$= \dfrac{(10+20i)(3+i)}{(3-i)(3+i)}$

$= \dfrac{30+70i-20}{10}$

$= 1+7i$

$z = 1+7i-4+2i$

$= -3+9i$

8. $i^4 = (i^2)^2 = (-1)^2 = 1$ TRUE

$\dfrac{1}{i^3} - i = \dfrac{i}{i^4} - i = \dfrac{i}{1} - i = 0$ TRUE

$\dfrac{1}{i} + i^3 = \dfrac{i^3}{i^4} + i^3 = \dfrac{i^3}{1} + i^3 = 2i^3 = -2i$ FALSE

$\dfrac{1}{i^2} = \dfrac{i^2}{i^4} = \dfrac{i^2}{1} = i^2$ TRUE

9. $(a+bi)^2 = 5+12i$

$a^2 + 2abi - b^2 = 5+12i$

Equating imaginary parts: $2ab = 12$

$b = \dfrac{6}{a}$

Equating real parts: $a^2 - b^2 = 5$

$a^2 - \dfrac{36}{a^2} = 5$

$a^4 - 36 = 5a^2$

$a^4 - 5a^2 - 36 = 0$

$(a^2-9)(a^2+4)$

Since a is real and positive, $a = 3$, and therefore $b = 2$.

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10. Let $z = x + iy$, so $z^* = x - iy$

$z + z^* = x + iy + x - iy = 2x$ so $z + z^*$ is real

$z - z^* = x + iy - (x - iy) = 2iy$ so $z - z^*$ is not real

$zz^* = (x + iy)(x - iy) = x^2 + y^2$ so zz^* is real

$\dfrac{z}{z^*} = \dfrac{z^2}{zz^*}$ zz^* is real but z^2 is not, so $\dfrac{z}{z^*}$ is complex.