Section 1: Further volumes of revolution

Section test

1. The volume of the solid obtained by rotating the curve $y=\frac{1}{\sqrt{x}}$ between $x = 1$ and $x = 2$ through 360° about the x-axis is

(a) $\pi \ln 2 - 1$
(b) $\pi(\ln 2 - 1)$
(c) $\pi \ln 2$
(d) $\pi(\ln 2 + 1)$

2. The volume of the solid obtained when the graph of $y=\sqrt{e^x - 1}$ between $x = 0$ and $x = 1$ is rotated through 360° about the x-axis is given by

(a) $\pi(e+1)$
(b) πe
(c) $\pi(e-1)$
(d) $\pi(e-2)$

3. The curve $y = \sec x$ between $x = 0$ and $x=\frac{\pi}{3}$ is rotated through 360° about the x-axis. The volume of the solid generated is

(a) $\frac{2\pi}{\sqrt{3}}$
(b) $\frac{\pi}{2}$
(c) $2\sqrt{3}\pi$
(d) $\sqrt{3}\pi$

4. The volume of the solid obtained by rotating the curve $y=\frac{1}{\sqrt{2x+1}}$ between $x = 0$ and $x = 2$ through 360° about the x-axis is

(a) $\frac{1}{2}\pi\left(\sqrt{5}-1\right)$
(b) $\pi \ln 5$
(c) $\frac{1}{2}\pi \ln 5$
(d) $\pi\left(\sqrt{5}-1\right)$

5. The volume of the solid obtained when the graph of $y = \ln(1 + x^2)$ between $y = 0$ and $y = \ln 2$ is rotated through 360° about the y-axis is

(a) $\int_0^1 \pi[\ln(1+x^2)]^2\,dx$
(b) $\pi\int_0^{\ln 2} \sqrt{(e^y - 1)}\,dy$
(c) $\int_0^{\ln 2} (e^y - 1)\,dy$
(d) $\pi\int_0^{\ln 2} (e^y - 1)\,dy$

6. The curve $y = \cos x$ between $x = 0$ and $x=\frac{\pi}{2}$ is rotated through 360° about the x-axis. The volume of the solid generated is

(a) $\frac{1}{4}\pi$
(b) $\frac{1}{4}\pi^2$
(c) $\frac{1}{2}\pi^2$
(d) $\frac{1}{4}\pi(\pi+1)$

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7. The curve $y = \dfrac{1}{\sqrt{x^2+1}}$ for $x > 0$ is rotated through 360° about the x-axis. Find the volume of the solid generated.

8. The part of the curve $x^4(1+y^2)=1$ between $y = 0$ and $y = 3$ is rotated through 360° about the y-axis. Find the volume of the solid generated.

9. A curve has parametric equations $x = t^3, y = t^2$. The region in the first quadrant bounded by the curve and the line $y = 4$ is rotated through 360° about the y-axis. Find the volume of the solid generated.

10. A curve has parametric equations $x = \mathrm{e}^{-2t}, y = \sqrt{1-t}$. The region bounded by the curve, the x-axis and the line $x = 1$ is rotated through 360° about the x-axis. The volume of the solid generated is

(a) $\frac{1}{2}\pi(1-\mathrm{e}^{-2})$ (b) $\frac{1}{4}\pi(1+\mathrm{e}^{-2})$

(c) $\frac{1}{2}\pi(1+\mathrm{e}^{-2})$ (d) $\frac{1}{4}\pi(1-\mathrm{e}^{-2})$

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Solutions to section test

1. $$\text{Volume} = \int_0^2 \pi y^2 dx = \pi \int_1^2 \left(\frac{1}{\sqrt{x}}\right)^2 dx$$
$$= \pi \int_1^2 \frac{1}{x} dx$$
$$= \pi [\ln x]_1^2$$
$$= \pi(\ln 2 - \ln 1)$$
$$= \pi \ln 2$$

2. $$\text{Volume} = \int_0^1 \pi y^2 dx$$
$$= \pi \int_0^1 (e^x - 1) dx$$
$$= \pi \left[e^x - x\right]_0^1$$
$$= \pi(e - 1 - 1 + 0)$$
$$= \pi(e - 2)$$

3. $$V = \int_0^{\frac{\pi}{3}} \pi y^2 \, dx$$
$$= \pi \int_0^{\frac{\pi}{3}} \sec^2 x \, dx$$
$$= \pi [\tan x]_0^{\frac{\pi}{3}}$$
$$= \pi \tan \tfrac{\pi}{3}$$
$$= \sqrt{3}\pi$$

4. $$\text{Volume} = \int_0^2 \pi y^2 dx = \pi \int_0^2 \left(\frac{1}{\sqrt{2x+1}}\right) dx$$
$$= \pi \int_0^2 \frac{1}{2x+1} dx$$
$$= \pi \left[\tfrac{1}{2} \ln(2x+1)\right]_0^2$$
$$= \tfrac{1}{2}\pi(\ln 5 - \ln 1)$$
$$= \tfrac{1}{2}\pi \ln 5$$

5. $$y = \ln(1 + x^2) \quad \Rightarrow e^y = 1 + x^2 \quad \Rightarrow x^2 = e^y - 1$$
$$\text{Volume} = \int_0^{\ln 2} \pi x^2 dy$$
$$= \pi \int_0^{\ln 2} (e^y - 1) dy$$

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6. $V=\int_0^{\frac{\pi}{2}} \pi y^2 \ dx$

$$
\begin{aligned}
&= \pi\int_0^{\frac{\pi}{2}} \cos^2 x \ dx \\
&= \pi\int_0^{\frac{\pi}{2}} \tfrac{1}{2}(1+\cos 2x) \ dx \\
&= \tfrac{1}{2}\pi\left[x+\tfrac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}} \\
&= \tfrac{1}{2}\pi\left(\tfrac{\pi}{2}+\tfrac{1}{2}\sin\pi-0\right) \\
&= \tfrac{1}{4}\pi^2
\end{aligned}
$$

7. $V=\int_0^{\infty} \pi y^2 \ dx$

$$
\begin{aligned}
&= \pi\int_0^{\infty} \frac{1}{x^2+1} \ dx \\
&= \pi\left[\arctan x\right]_0^{\infty} \\
&= \pi\times\frac{\pi}{2} \\
&= \frac{\pi^2}{2} = 4.93 \ (3 \text{ s.f.})
\end{aligned}
$$

8. $V=\int_0^{3} \pi x^2 \ dy$

$$
\begin{aligned}
&= \pi\int_0^{3} \frac{1}{\sqrt{1+y^2}} \ dx \\
&= \pi\left[\ln\left(y+\sqrt{y^2+1}\right)\right]_0^{3} \\
&= \pi\left(\ln(3+\sqrt{10})-\ln 1\right) \\
&= 5.71 \ (3 \text{ s.f.})
\end{aligned}
$$

9. $x=t^3, y=t^2$

When $y=4, t=2$ (t must be positive as x is positive in the first quadrant)

$$
\begin{aligned}
\text{Volume } &= \pi\int_{t=0}^{t=2} x^2 \frac{dy}{dt} dt \\
&= \pi\int_0^{2} t^6 \times 2t \ dt \\
&= \pi\int_0^{2} 2t^7 \ dt \\
&= \pi\left[\tfrac{1}{4}t^8\right]_0^{2} \\
&= 64\pi
\end{aligned}
$$

10. When the curve meets the x-axis, $y = 0 \quad \Rightarrow \sqrt{1-t} = 0 \quad \Rightarrow t = 1$

When $x = 1, \quad e^{-2t} = 1 \Rightarrow t = 0$

$$x = e^{-2t} \Rightarrow \frac{dx}{dt} = -2e^{-2t}$$

$$\text{Volume } = \int_{t=1}^{t=0} \pi y^2 \frac{dx}{dt} dt$$

$$= \pi \int_1^0 (1-t) \times -2e^{-2t} dt$$

$$= -2\pi \int_1^0 (1-t)e^{-2t} dt$$

Using integration by parts:

Let $u = 1 - t \Rightarrow \frac{du}{dx} = -1$

$$\frac{dv}{dx} = e^{-2x} \Rightarrow v = -\tfrac{1}{2}e^{-2x}$$

$$\int_1^0 (1-t)e^{-2t} dt = \left[(1-t) \times -\tfrac{1}{2}e^{-2t}\right]_1^0 - \int_{-1}^{1} -1 \times -\tfrac{1}{2}e^{-2t} dt$$

$$= \left[-\tfrac{1}{2}(1-t)e^{-2t}\right]_1^0 - \int_{-1}^{1} \tfrac{1}{2}e^{-2t} dt$$

$$= \left[-\tfrac{1}{2}(1-t)e^{-2t} + \tfrac{1}{4}e^{-2t}\right]_1^0$$

$$= \left(-\tfrac{1}{2} + \tfrac{1}{4}\right) - \left(0 + \tfrac{1}{4}e^{-2}\right)$$

$$= -\tfrac{1}{4} - \tfrac{1}{4}e^{-2}$$

$$\text{Volume } = -2\pi(-\tfrac{1}{4} - \tfrac{1}{4}e^{-2})$$

$$= \tfrac{1}{2}\pi(1 + e^{-2})$$