Section 2: Mean values and general integration

Section test

1. Find the mean value of the function $y = x^3$ in the interval [0, 4].

2. Find the mean value of the function $y = \sqrt{x}$ in the interval [1, 4].

3. Find the mean value of the function $\mathrm{f}(x) = \cosh x$ between $x = 0$ and $x = 2$.

4. Find the mean value of the function $\mathrm{f}(x) = \dfrac{1}{\sqrt{x^2 - 1}}$ between $x = 1$ and $x = 1.25$.

5. Which of the following integrals give a result involving an arsinh function?

(a) $\displaystyle\int \frac{1}{\sqrt{2x^2 - 3}}\,\mathrm{d}x$ (b) $\displaystyle\int \frac{1}{\sqrt{2x^2 + 3}}\,\mathrm{d}x$

(c) $\displaystyle\int \frac{1}{\sqrt{3 - 2x^2}}\,\mathrm{d}x$ (d) $\displaystyle\int \frac{1}{\sqrt{x^2 - 2x + 4}}\,\mathrm{d}x$

(e) $\displaystyle\int \frac{1}{\sqrt{x^2 - 2x}}\,\mathrm{d}x$

6. Which of the following integrals give a result involving an arcosh function?

(a) $\displaystyle\int \frac{1}{\sqrt{2x^2 - 3}}\,\mathrm{d}x$ (b) $\displaystyle\int \frac{1}{\sqrt{2x^2 + 3}}\,\mathrm{d}x$

(c) $\displaystyle\int \frac{1}{\sqrt{3 - 2x^2}}\,\mathrm{d}x$ (d) $\displaystyle\int \frac{1}{\sqrt{x^2 - 2x + 4}}\,\mathrm{d}x$

(e) $\displaystyle\int \frac{1}{\sqrt{x^2 - 2x}}\,\mathrm{d}x$

7. Which of the following would be an appropriate substitution to use in order to find the integral $\displaystyle\int \sqrt{a^2 + x^2}\,\mathrm{d}x$?

(a) $x = a\sin u$ (b) $x = a\cos u$ (c) $x = a\tan u$
(d) $x = a\sinh u$ (e) $x = a\cosh u$

8. Which of the following would be an appropriate substitution to use in order to find the integral $\displaystyle\int \frac{1}{(a^2 - x^2)^{\frac{3}{2}}}\,\mathrm{d}x$?

(a) $x = a\sin u$ (b) $x = a\cos u$ (c) $x = a\tan u$
(d) $x = a\sinh u$ (e) $x = a\cosh u$

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Solutions to section test

1. Mean value $= \frac{1}{4-0}\int_0^4 x^3 dx$

$= \frac{1}{4}\left[\frac{1}{4}x^4\right]_0^4$

$= \frac{1}{4}\times\frac{1}{4}\times 4^4$

$= 16$

2. Mean value $= \frac{1}{4-1}\int_1^4 x^{\frac{1}{2}} dx$

$= \frac{1}{3}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^4$

$= \frac{2}{9}(8-1)$

$= \frac{14}{9}$

3. Mean $= \frac{1}{2}\int_0^2 \cosh x\, dx$

$= \frac{1}{2}[\sinh x]_0^2$

$= \frac{1}{2}\sinh 2$

$= \frac{e^2 - e^{-2}}{4}$

$= 1.81$ (3 s.f.)

4. Mean $= \frac{1}{0.25}\int_1^{1.25} \frac{1}{\sqrt{x^2-1}} dx$

$= 4\left[\ln\left(x+\sqrt{x^2-1}\right)\right]_1^{1.25}$

$= 4\ln(1.25+0.75) - 4\ln 1$

$= 4\ln 2$

5. $\int \frac{1}{\sqrt{2x^2-3}} dx$ is not – this is the form for a arcosh function

$\int \frac{1}{\sqrt{2x^2+3}} dx$ does give a result which is a arsinh function

$\int \frac{1}{\sqrt{3-2x^2}} dx$ is not – this is the form for a arcsin function

$\int \frac{1}{\sqrt{x^2-2x+4}} dx = \int \frac{1}{\sqrt{(x-1)^2+3}} dx$ so this does give a result which is a arsinh function

$\int \frac{1}{\sqrt{x^2-2x}}dx = \int \frac{1}{\sqrt{(x-1)^2-1}}dx$ so this is the form for a arcosh function

6. From above, the integrals $\int \frac{1}{\sqrt{2x^2-3}}dx$ and $\int \frac{1}{\sqrt{x^2-2x}}dx = \int \frac{1}{\sqrt{(x-1)^2-1}}dx$ give arcosh functions

7. $x = a\tan u$ could be used giving $\sqrt{a^2+a^2\tan^2 u} = \sqrt{a^2\sec^2 u} = a\sec u$
$x = a\sinh u$ could be used giving $\sqrt{a^2+a^2\sinh^2 u} = \sqrt{a^2\cosh^2 u} = a\cosh u$

8. $x = a\sin u$ could be used giving $\frac{1}{(a^2-a^2\sin^2 u)^{\frac{3}{2}}} = \frac{1}{(a^2\cos^2 u)^{\frac{3}{2}}} = \frac{1}{a^2}\sec^3 u$
$x = a\cos u$ could be used giving $\frac{1}{(a^2-a^2\cos^2 u)^{\frac{3}{2}}} = \frac{1}{(a^2\sin^2 u)^{\frac{3}{2}}} = \frac{1}{a^2}\operatorname{cosec}^3 u$