Topic assessment

1. A solution is sought to the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}+2y=2\mathrm{e}^{-x}$$

(i) Find the general solution. [8]

You are given that when $x = 0$, $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$.

(ii) Find the solution subject to these conditions. [3]

2. (i) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3\frac{\mathrm{d}y}{\mathrm{d}x}+2y=2x-1.$$ [8]

(ii) Find the particular solution for which $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$. [3]

3. (i) Find the general solution to the differential equation $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+9y=18$. [8]

(ii) Given that y has a maximum value of 6 when $x=\frac{\pi}{2}$, find the minimum value of y. [4]

(iii) Find the smallest positive value of x for which $y = 0$. [2]

4. A system of differential equations is given by

$$\frac{\mathrm{d}x}{\mathrm{d}t}=-3x-2y-4 \qquad \frac{\mathrm{d}y}{\mathrm{d}t}=x-y+3$$

and when $t = 0$, $x = 0$ and $y = 3$.

(i) Find expressions for x and y in terms of t. [10]

(ii) Describe what happens to x and y as t tends to ∞. [1]

5. A particle is attached to the lower end of a spring, the upper end of which oscillates about a point O. The motion of the particle can be modelled by the equation

$$\ddot{x}+25x=0.5\sin 5t$$

where x is the displacement of the particle from its equilibrium point.
When $t = 0$, $x = 0$ and the particle is at rest.

(i) Solve this differential equation to find x in terms of t and describe briefly the motion of the particle. [10]

In order to damp the oscillations the particle is submerged in liquid and the motion of the particle can be modelled as

$$\ddot{x}+k\dot{x}+25x=0.5\sin 5t$$

where k is a constant.

(ii) Explain why k must be positive. Give the range of values of k for which the system will be underdamped. [3]

Total 60 marks