

Topic assessment

1. (i) Sketch the polar curve whose equation is

$$r = 3\sin 2\theta,$$
 dotting in parts of the curve corresponding to $r < 0$.
 Indicate clearly the order in which the various parts of the curve are traced out as θ increases from 0 to 2π .

[5]

- (ii) Find the area of the loop in the first quadrant.

[5]

2. (i) Sketch the curve (a *cardioid*) whose equation in polar coordinates is

$$r = a(1 + \cos \theta),$$
 where a is a positive constant.

[5]

- (ii) Show that the cardioid meets the circle $r = a$ when $\theta = \pm \frac{1}{2}\pi$.
 Sketch the circle $r = a$ on the same diagram as the cardioid.

[4]

- (iii) Shade the region with area given by $\int_{\frac{1}{2}\pi}^{\pi} \frac{1}{2}r^2 d\theta$ where $r = a(1 + \cos \theta)$.

[2]

- (iv) Find the area of the region which is both inside the cardioid and inside the circle.

[9]

3. (r, θ) are polar coordinates with origin O.

- (i) Sketch the curve with equation $r = k\theta$ for $0 \leq \theta \leq 4\pi$, where k is a positive constant. Label the points A and B on the curve corresponding to $\theta = \pi$ and $\theta = 2\pi$ respectively.

[5]

- (ii) On your diagram, shade in the region bounded by the line AOB and that part of the curve $r = k\theta$ for which $\pi \leq \theta \leq 2\pi$.

[2]

- (iii) The area of the shaded region is S_1 . Calculate S_1 .

[8]

- (iv) S_2 is the area of a semicircle with diameter AB.

Calculate the value of $\frac{S_1}{S_2}$.

[5]

Total: 50 marks