

Topic assessment

1. (i) For the function $f(x) = e^{3x} \cos 2x$, express $f'(x)$ in terms of $f(x)$. [3]
(ii) Express $f''(x)$ in terms of $f'(x)$ and $f(x)$. [3]
(iii) Express $f^{(3)}(x)$ in terms of $f''(x)$, $f'(x)$ and $f(x)$. [3]
(iv) Hence find the first four non-zero terms in the Maclaurin series for $f(x) = e^{3x} \cos 2x$. [4]

2. Use known Maclaurin series to find the first three non-zero terms in the Maclaurin series for each of the following functions.
 - (i) $\sin x^2$ [3]
 - (ii) $\cos^2 x$ [5]
 - (iii) $\ln(1 + \sin x)$ [5]

3. (i) Differentiate $\arcsin 3x$. [2]
(ii) Find the first three terms in the series expansion of $\frac{1}{\sqrt{1-9x^2}}$ [5]
(iii) Hence find the first three terms in the series expansion of $\arcsin 3x$. [3]

4. (i) Find the first four terms in the expansion of $\ln(1+4x)$. [4]
(ii) Find the first four terms in the expansion of $\ln(1-3x)$. [4]
(iii) Hence find the first four terms in the expansion of $\ln\left(\frac{1+4x}{1-3x}\right)$ [2]
(iv) For what values of x is this expansion valid? [1]
(v) By substituting $x = 0.1$, use your answer to (iii) to find an approximate value for $\ln 2$. Give your answer to an appropriate degree of accuracy. [3]

Total 50