Topic assessment

1. If possible find the values of

   (i) $\int_2^{\infty} x^{-3}\,dx$ [4]

   (ii) $\int_{-3}^{\infty} x^{-4}\,dx$ [4]

   (iii) $\int_{-1}^{8} x^{-\frac{1}{3}}\,dx$ [4]

   (iv) $\int_0^{49} \frac{1}{\sqrt{x}}\,dx$ [4]

2. Use the substitution $x\sqrt{3} = 2\tan\theta$ to show that $\int_0^2 \frac{1}{(3x^2+4)^{\frac{3}{2}}}\,dx = \frac{1}{8}$. [6]

3. (i) Find the exact value of $\int_0^1 \frac{1}{\sqrt{4-3x^2}}\,dx$. [5]

   (ii) Solve the differential equation $\frac{dy}{dx} = 1+4y^2$, given that $y = 0$ when $x = 4$.

   Express y in terms of x. [6]

4. (i) Differentiate $\arcsin\left(\frac{x}{2}\right)$ with respect to x (where $0 < x < 2$), simplifying your answer as much as possible. [3]

   (ii) Using integration by parts, show that $\int_0^{\sqrt{3}} \arcsin\left(\frac{x}{2}\right)dx = \frac{\pi}{\sqrt{3}} - 1$. [6]

5. (i) Express $\frac{2x}{(1+x)(1+x^2)}$ in partial fractions. [4]

   (ii) Hence evaluate $\int_0^1 \frac{2x}{(1+x)(1+x^2)}\,dx$, expressing your answer in an exact form. [4]

Total 50 marks