Edexcel Further Maths First order DEs

Topic assessment

1. (i) Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = y - \frac{1}{y} \qquad (-1 < y < 1)$$ [5]

(ii) Sketch the family of solution curves represented by the general solution. [3]

(iii) Find the particular solution for which $y = 0$ when $x = 1$, and indicate this particular solution on your sketch. [3]

2. (i) Solve the differential equation

$$(1+x^2)\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{4x^3y}{1-x^2} = 1 \qquad (-1 < x < 1)$$

giving y in terms of x. [8]

(ii) Find the particular solution in the case where $y = 1$ when $x = 0$. [2]

3. The motion of a parachutist free-falling from rest from a stationary helicopter is given by the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = 9.8 - 0.002v^2$$

where x m is her distance below the helicopter and v ms^{-1} is her velocity.
Solve the differential equation to show that $v = 70(1-\mathrm{e}^{-0.004x})^{1/2}$. [6]

4. Two differential equations are being studied for $x > 0$.

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x} = \mathrm{e}^x \qquad ①$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\mathrm{e}^x = \mathrm{e}^x \qquad ②$$

(i) Use the integrating factor method to find the general solution of equation ①, giving y in terms of x. [6]

(ii) Given that $y = 1$ when $x = 1$, find the particular solution. [2]

(iii) Use a method other than the integrating factor method to find the general solution of equation ②, giving y in terms of x. [5]

Total: 40 marks