Section 1: The Normal distribution

Section test

1. Assume that the masses of adult men can be modelled by the Normal distribution with mean 75 kg and standard deviation 5 kg.

   What is the probability that an adult man, chosen at random, will have a mass greater than 77.5 kg?

   What is the probability that an adult man, chosen at random, will have a mass between 76.6 kg and 83.5 kg?

   62% of adult men have a mass greater than M kg. What is M?

   What is the interquartile range for the masses of adult men?

2. A manufacturer produces bolts, the internal diameter of which can be modelled by the Normal distribution $N(2.8, 0.2^2)$ (the mean and standard deviation are measured in mm).

   What percentage of bolts have an internal diameter measurement within one standard deviation of the mean?

   Bolts which have an internal diameter of less than 2.7 mm or greater than 2.92 mm are rejected. Out of a batch of 800 bolts, how many would be acceptable (to the nearest whole number)?

3. The results of an examination, in which there were 2454 candidates, are modelled by a Normal distribution with mean 62 and standard deviation 15. Marks are recorded as integers.

   If the pass mark is 40, approximately how many candidates would you expect to pass?

   Approximately how many candidates (to the nearest 5) would you expect to gain marks between 50 and 70 inclusive?

4. Assume that the lifetime of a certain type of light bulb can be modelled by the Normal distribution. It is found that 6% of bulbs last for more than 1100 hours and 14% for less than 970 hours.

   What is the mean lifetime of a light bulb?

   What is the standard deviation of the lifetime of a light bulb?

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Solutions to section test

1. $X \sim N(75, 5^2)$
   Using a calculator function, $P(X > 77.5) = 0.3085$

   $P(76.6 < X < 83.5) = 0.3299$

   $P(X > M) = 0.62$
   $P(X < M) = 0.38$
   Using the inverse normal function on a calculator, $M = 73.5$ (3 s.f.)

   Let upper quartile be Q_3
   $P(X < Q_3) = 0.75$
   $Q_3 = 78.35$ (3 s.f.)
   Distance of upper quartile from mean = 3.35
   By symmetry, interquartile range is twice this
   so interquartile range $= 6.7$

2. $X \sim N(2.8, 0.2^2)$
   $P(2.6 < X < 3) = 0.6827$
   The percentage of bolts within one standard deviation of the mean is 68.3%.

   $P(2.7 < X < 2.92) = 0.4172$
   Number acceptable $= 0.4172 \times 800 = 334$ (to nearest whole number)

3. Let X be the number of candidates who pass
   $X \sim N(62, 15^2)$
   The pass mark is 40, so the probability of passing is $P(X \geq 39.5)$.
   $P(X \geq 39.5) = 0.9332$
   Expected number of passes $= 0.9332 \times 2454$
   $= 2290$ (to nearest whole number)

   Probability of gaining marks between 50 and 70 inclusive is
   $P(49.5 \leq X < 70.5) = 0.5124$
   Expected number of candidates in this range is approx. $0.5124 \times 2454 = 1257$
   To the nearest 5, this is 1255 candidates.

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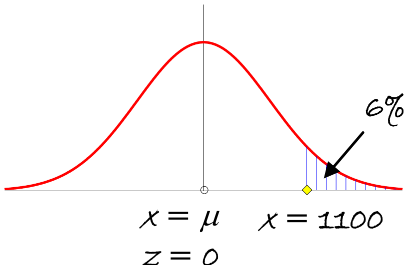
4. $P(X > 1100) = 0.06$

$P(X < 1100) = 0.94$

$\Phi(z_1) = 0.94$

$z_1 = 1.555$

$\dfrac{1100 - \mu}{\sigma} = 1.555$

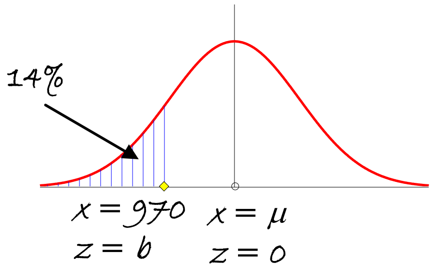


$P(X \leq 970) = 0.14$

$\Phi(z_2) = 0.14$

$z_2 = -1.080$

$\dfrac{970 - \mu}{\sigma} = -1.080$



$1100 - \mu = 1.555\sigma$

$970 - \mu = -1.080\sigma$

Subtracting: $130 = 2.635\sigma$

$\sigma = 49.3$

$\mu = 970 + 1.080\sigma = 1023$

The mean lifetime of a light bulb is 1023 hours (to nearest whole number)

The standard deviation is 49.3 hours.