**9MA0/01: Pure Mathematics Paper 1 Mark scheme**

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **1 (a)** | |  | B1 | 1.1b |
| M1 | 1.1b |
| (3 dp) | A1 | 1.1b |
|  | **(3)** |  |
| **(b)** | | Any valid reason, for example   * Increase the number of strips * Decrease the width of the strips * Use more trapezia between  and | B1 | 2.4 |
|  | **(1)** |  |
| **(c)(i)** | |  | B1ft | 2.2a |
| **(c)(ii)** | |  | B1ft | 2.2a |
|  | |  | **(2)** |  |
| **(6 marks)** | | | | |
| **Question 1 Notes:** | | | | |
| **(a)** |  | | | |
| **B1:** | Outside brackets  or  or 0.25 or | | | |
| **M1:** | For structure of trapezium rule.  No errors are allowed, e.g. an omission of a *y*-ordinate or an extra *y*-ordinate or a repeated *y*-ordinate. | | | |
| **A1:** | Correct method leading to a correct answer only of 1.635 | | | |
| **(b)** |  | | | |
| **B1:** | See scheme | | | |
| **(c)** |  | | | |
| **B1:** | or a value which is 5  their answer to part (a) | | | |
|  | **Note:** Allow B1ft for 8.176 (to 3 dp) which is found from | | | |
|  | **Note:** Do not allow an answer of 8.1886… which is found directly from integration | | | |
| **(d)** |  | | | |
| **B1:** | 13.635 or a value which is 12 + their answer to part (a) | | | |
|  | **Note:** Do not allow an answer of 13.6377… which is found directly from integration | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **2 (a)** |  | B1 | 1.1b |
|  | M1 | 1.1b |
| A1ft | 1.1b |
|  | A1 | 2.1 |
|  | **(4)** |  |
| **(b)(i)** |  | M1 | 1.1b |
|  |  |  |
| or | M1 | 3.1a |
| So,  or | A1 | 1.1b |
| **(b)(ii)** | satisfies  (o.e.), so the approximation is valid. | B1 | 2.3 |
|  | **(4)** |  |
| **(8 marks)** | | | |

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| **Question 2 Notes:** | |
| **(a)** |  |
| **B1:** | Manipulates  by taking out a factor of or 2 |
| **M1:** | Expands  to give at least 2 terms which can be simplified or un-simplified, |
|  | E.g.  or  or |
|  | where  is a numerical value and **where** . |
| **A1ft:** | A correct simplified or un-simplified expansion with **consistent** |
| **A1:** | Fully correct solution leading to  where |
| **(b)(i)** |  |
| **M1:** | Attempts to substitute  or 0.1 into |
| **M1:** | A complete method of finding an approximate value for  E.g.   * substituting  or 0.1 into their part (a) binomial expansion and equating the result to an expression of the form  or * followed by re-arranging to give |
| **A1:** | **or any equivalent fraction**, e.g. or |
|  | Also allow  **or any equivalent fraction** |
| **(b)(ii)** |  |
| **B1:** | Explains that the approximation is valid because  satisfies |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **3 (a)** | |  | M1 | 1.1b |
|  | M1 | 2.2a |
|  | A1 | 1.1b |
|  | **(3)** |  |
| **(b)** | |  | B1ft | 2.2a |
|  | |  | **(1)** |  |
| **(4 marks)** | | | | |
| **Question 3 Notes:** | | | | |
| **(a)** |  | | | |
| **M1:** | Uses the formula  with  to generate values for  and | | | |
| **M1:** | Finds  and deduces | | | |
| **A1:** | which leads to a correct answer of | | | |
| **(b)** |  | | | |
| **B1ft:** | Follow through on their periodic function. Deduces that either | | | |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **4 (a)** | | , , |  |  |
| or | M1 | 3.1a |
| So | A1 | 1.1b |
|  | **(2)** |  |
| **(b)** | |  | M1 | 1.1b |
| As  then |  |  |
| or | M1 | 3.1a |
| So  only | A1 | 1.1b |
|  | **(3)** |  |
| **(5 marks)** | | | | |
| **Question 4 Notes:** | | | | |
| **(a)** |  | | | |
| **M1:** | A complete method for finding the position vector of *D* | | | |
| **A1:** | or | | | |
| **(b)** |  | | | |
| **M1:** | A complete attempt to find  or | | | |
| **M1:** | A complete process for finding the position vector of *X* | | | |
| **A1:** | or | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5 (a)(i)** |  |  |  |
|  | M1 | 1.1b |
|  | A1\* | 1.1b |
| **(a)(ii)** | Hence, | M1 | 2.2a |
| A1 | 1.1b |
|  | **(4)** |  |
| **(b)** |  |  |  |
| E.g. | M1 | 1.2 |
|  | M1 | 1.1b |
|  | B1 | 1.1b |
|  |  |  |
| \* | A1 \* | 2.1 |
|  | **(4)** |  |
| **(c)** |  |  |  |
|  |  |  |
| Reason 1: E.g.   * is not defined when * is not defined when * but  is only defined for   Reason 2:   * so  has no (real) roots |  |  |
| At least one of Reason 1 or Reason 2 | B1 | 2.4 |
| Both Reason 1 and Reason 2 | B1 | 2.1 |
|  | **(2)** |  |
| **(10 marks)** | | | |

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| **Question 5 Notes:** | |
| **(a)(i)** |  |
| **M1:** | Applies |
| **A1\*:** | Applies  to show that |
| **(a)(ii)** |  |
| **M1:** | Deduces  is a factor of  and attempts to find a quadratic factor of by either equating coefficients or by algebraic long division |
| **A1:** |  |
| **(b)** |  |
| **M1:** | Evidence of applying a correct law of logarithms |
| **M1:** | Uses correct laws of logarithms to give either |
|  | * an expression of the form  where *k* is a constant * an expression of the form |
| **B1:** | Evidence in their working of 8 |
| **A1\*:** | Correctly proves  with no errors seen |
| **(c)** |  |
| **B1:** | See scheme |
| **B1:** | See scheme |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **6 (a)** | | Attempts to use an appropriate model;  e.g. | M1 | 3.3 |
| e.g.  Substitutes | M1 | 3.1b |
| or | A1 | 1.1b |
|  | **(3)** |  |
| **(b)** | | Substitutes  into their | M1 | 3.4 |
| Coach can enter the tunnel | A1 | 2.2b |
|  | **(2)** |  |
| **(b)**  **Alt 1** | | so maximum width | M1 | 3.4 |
| Coach can enter the tunnel | A1 | 2.2b |
|  | **(2)** |  |
| **(c)** | | E.g.   * Coach needs to enter through the centre of the tunnel. This will only be possible if it is a one-way tunnel * In real-life the road may be cambered (and not horizontal) * The quadratic curve *BCA* is modelled for the entrance to the tunnel but we do not know if this curve is valid throughout the entire length of the tunnel * There may be overhead lights in the tunnel which may block the path of the coach | B1 | 3.5b |
|  | **(1)** |  |
| **(6 marks)** | | | | |
| **Question 6 Notes:** | | | | |
| **(a)** |  | | | |
| **M1:** | Translates the given situation into an appropriate quadratic model – see scheme | | | |
| **M1:** | Applies the maximum height constraint in an attempt to find the equation of the model – see scheme | | | |
| **A1:** | Finds a suitable equation – see scheme | | | |
| **(b)** |  | | | |
| **M1:** | See scheme | | | |
| **A1:** | Applies a fully correct argument to infer {by assuming that curve *BCA* is quadratic and the given measurements are correct}, that is possible for the coach to enter the tunnel | | | |
| **(c)** |  | | | |
| **B1:** | See scheme | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7** | , |  |  |
|  | M1 | 3.1a |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | M1 | 2.2a |
|  | A1 | 2.1 |
|  | **(5)** |  |
|  |  |  |  |
| **7**  **Alt 1** | , |  |  |
|  | M1 | 3.1a |
| M1 | 1.1b |
|  | A1 | 1.1b |
|  | M1 | 2.2a |
|  | A1 | 2.1 |
|  | **(5)** |  |
| **(5 marks)** | | | |

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| **Question 7 Notes:** | |
| **M1:** | Attempts to solve the problem by recognising the need to apply a method of integration by parts on either  or . Allow this mark for either       where  are constants. |
| **M1:** | For either |
| **A1:** | Correct integration which can be simplified or un-simplified. E.g. |
| **M1:** | Deduces that the upper limit is 2 and uses limits of 2 and 0 on their integrated function |
| **A1:** | Correct proof leading to  where |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **8 (a)** | | Total amount | M1 | 3.1b |
|  | A1 | 1.1b |
|  | **(2)** |  |
| Total Cost = | M1 | 3.1b |
| M1 | 1.1b |
|  |  |  |
| (nearest £1000) | A1 | 3.2a |
|  | **(3)** |  |
| **(5 marks)** | | | | |
| **Question 8 Notes:** | | | | |
| **(a)** |  | | | |
| **M1:** | Attempts to apply the correct geometric summation formula with either , | | | |
|  | and  (Condone | | | |
| **A1:** | Correct answer of 31800 (tonnes) | | | |
| **(b)** |  | | | |
| **M1:** | Fully correct method to find the total cost | | | |
| **M1:** | For either | | | |
| **A1:** | Correct answer of £169000  **Note:** Using rounded answer in part (a) gives 168710 which becomes £169000 (nearest £1000) | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **9** | Gradient of chord | B1 | 1.1b |
| M1 | 2.1 |
|  | B1 | 1.1b |
| Gradient of chord = |  |  |
|  |  |  |
|  |  |  |
|  | A1 | 1.1b |
| and so at *P*, | A1 | 2.2a |
|  | **(5)** |  |
| **9**  **Alt 1** | Let a point *Q* have *x* coordinate  so | B1 | 1.1b |
|  |  |  |
| Gradient *PQ* = | M1 | 2.1 |
|  | B1 | 1.1b |
| Gradient *PQ* = |  |  |
|  |  |  |
|  |  |  |
|  | A1 | 1.1b |
|  | A1 | 2.2a |
|  | **(5)** |  |
| **(5 marks)** | | | |

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| **Question 9 Notes:** | |
| **B1:** | seen or implied |
| **M1:** | Begins the proof by attempting to write the gradient of the chord in terms of *x* and *h* |
| **B1:** | by expanding brackets or by using a correct binomial expansion |
| **M1:** | Correct process to obtain the gradient of the chord as |
| **A1:** | Correctly shows that the gradient of the chord is  and applies a limiting argument to deduce when   Finally, deduces that at the point *P*, |
|  | **Note:** can be used in place of *h* |
| **Alt 1** |  |
| **B1:** | Writes down the *y* coordinate of a point close to *P*.  E.g. For a point *Q* with |
| **M1:** | Begins the proof by attempting to write the gradient of the chord *PQ* in terms of *h* |
| **B1:** | by expanding brackets or by using a correct binomial expansion |
| **M1:** | Correct process to obtain the gradient of the chord *PQ* as |
| **A1:** | Correctly shows that the gradient of *PQ* is and applies a limiting argument to deduce that at the point  on |
|  | **Note:** For Alt 1, can be used in place of *h* |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **10 (a)** |  | M1 | 1.1b |
|  | M1 | 2.1 |
| Hence | A1 | 2.5 |
|  | **(3)** |  |
| **(b)** |  | M1 | 1.1a |
|  | M1 | 1.1b |
| A1 | 1.1b |
| (note that ) | A1 | 2.1 |
|  | **(4)** |  |
| **(c)** |  | M1 | 1.1b |
| A1 | 1.1b |
|  | **(2)** |  |
| **(d)** | . Hence | M1 | 2.1 |
| Either  or  or | B1 | 1.1b |
| or | A1 | 1.1b |
|  | **(3)** |  |
| **(e)** | E.g.   * the function g is many-one * the function g is not one-one * the inverse is one-many | B1 | 2.4 |
|  | **(1)** |  |
| **(13 marks)** | | | |

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| **Question 10 Notes:** | |
| **(a)** |  |
| **M1:** | Attempts to find the inverse by cross-multiplying and an attempt to collect all the *x-*terms (or swapped *y*-terms) onto one side |
| **M1:** | A fully correct method to find the inverse |
| **A1:** | A correct  , expressed fully in function notation (including the domain) |
| **(b)** |  |
| **M1:** | Attempts to substitute  into |
| **M1:** | Applies a method of “rationalising the denominator” for both their numerator and their denominator. |
| **A1:** | which can be simplified or un-simplified |
| **A1:** | Shows  where  or  with no errors seen. |
| **(c)** |  |
| **M1:** | Attempts to substitute the result of  into f |
| **A1:** | Correctly obtains |
| **(d)** |  |
| **M1:** | Full method to establish the minimum of g.  E.g.   * leading to * Finds the value of *x* for which  and inserts this value of *x* back into  in order to find to |
| **B1:** | For either   * finding the correct minimum value of g   (Can be implied by  or )   * stating |
| **A1:** | States the correct range for g. E.g.  or |
| **(e)** |  |
| **B1:** | See scheme |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **11 (a)** |  |  |  |
|  | M1 | 1.1b |
| **Criteria 1**  **Either**    **or**    **Criteria 2**  **Either**      **or** |  |  |
| At least one of Criteria 1 or Criteria 2 | B1 | 2.4 |
| Both Criteria 1 and Criteria 2  **and** concludes *C* has a point of inflection at | A1 | 2.1 |
|  | **(3)** |  |
| **(b)** |  |  |  |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | A1 | 2.2a |
|  | M1 | 2.1 |
|  | A1 | 1.1b |
|  | M1 | 2.1 |
| So, | A1 | 1.1b |
|  | **(7)** |  |
| **(10 marks)** | | | |

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| **Question 11 Notes:** | |
| **(a)** |  |
| **M1:** | E.g.   * attempts to find * finds  and sets the result equal to 0 |
| **B1:** | See scheme |
| **A1:** | See scheme |
| **(b)** |  |
| **M1:** | Integrates  to give ,  with or without the constant of integration |
| **A1:** | , with or without the constant of integration |
| **A1:** | Finds , and makes some reference to  passing through the origin to deduce  Proceeds to produce the result  or |
| **M1:** | Uses a valid method to solve the quadratic equation to give *x* in terms of *k* |
| **A1** | Correct roots for *x* in terms of *k*. i.e. |
| **M1:** | Applies on in a complete method to find |
| **A1:** | Finds  from correct solution only |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **12** |  |  |  |
| Attempts this question by applying the substitution  and progresses as far as achieving | M1 | 3.1a |
| and | M1 | 1.1b |
|  | A1 | 2.1 |
|  | M1 | 1.1b |
| M1 | 1.1b |
|  | M1 | 1.1b |
|  | A1\* | 2.1 |
|  | **(7)** |  |
| **12**  **Alt 1** | Attempts this question by applying the substitution  and progresses as far as achieving | M1 | 3.1a |
| and | M1 | 1.1b |
|  | A1 | 2.1 |
|  | M1 | 1.1b |
| M1 | 1.1b |
|  | M1 | 1.1b |
|  | A1\* | 2.1 |
|  | **(7)** |  |
| **(7 marks)** | | | |

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| **Question 12 Notes:** | |
| **M1:** | See scheme |
| **M1:** | Attempts to differentiate  to give  and applies |
| **A1:** | Applies to show that the integral becomes |
| **M1:** | Achieves an expression in *u* that can be directly integrated (e.g. dividing each term by *u* or applying partial fractions) and integrates to give an expression in *u* of the form |
| **M1:** | For integration in *u* of the form |
| **M1:** | Applies *u-*limits of 1 and 2 to an expression of the form  and subtracts either way round |
| **A1\*:** | Applies *u*-limits the right way round, i.e.       and correctly proves  with no errors seen |
| **Alt 1** |  |
| **M1:** | See scheme |
| **M1:** | Attempts to differentiate  to give  and applies |
| **A1:** | Applies  to show that the integral becomes |
| **M1:** | Achieves an expression in *u* that can be directly integrated (e.g. by applying partial fractions or a substitution ) and integrates to give an expression in *u* of the form  or  where |
| **M1:** | For integration in *u* in the form |
| **M1:** | Either   * Applies *u-*limits of 0 and 1 to an expression of the form  and subtracts either way round * Applies  of 1 and 2 to an expression of the form  where  and subtracts either way round |
| **A1\*:** | Applies *u*-limits the right way round, (o.e. in ) i.e.       and correctly proves  with no errors seen |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **13 (a)** |  | B1 | 1.1b |
| o.e. | M1 | 1.1b |
| so | A1 | 1.1b |
|  | **(3)** |  |
| **(b)** | e.g.  or | B1 | 3.4 |
|  | **(1)** |  |
| **(c)** |  | B1ft | 3.4 |
|  | **(1)** |  |
| **(d)** | Sets | M1 | 1.1b |
| Afternoon solution | M1 | 3.1b |
| or 4:54 pm | A1 | 3.2a |
|  | **(3)** |  |
| **(e)(i)** | * An attempt to find the depth of water at 00:00 on 19th October 2017 for at least one of either Tom’s model or Jolene’s model. | M1 | 3.4 |
| * At 00:00 on 19thOctober 2017,   Tom:  and Jolene:  and e.g.   * As  then Jolene’s model is not true * Jolene’s model is not continuous at 00:00 on 19th October 2017 * Jolene’s model does not continue on from where Tom’s model has ended | A1 | 3.5a |
| **(ii)** | To make the model continuous, e.g. | B1 | 3.3 |
|  | **(3)** |  |
| **(11 marks)** | | | |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **13 (d)**  **Alt 1** | | Sets | M1 | 1.1b |
| Afternoon solution | M1 | 3.1b |
| or 4:54 pm | A1 | 3.2a |
|  | **(3)** |  |
| **Question 13 Notes:** | | | | |
| **(a)** |  | | | |
| **B1:** | Condone | | | |
| **M1:** | For either | | | |
| **A1:** |  | | | |
| **(b)** |  | | | |
| **B1:** | Uses Tom’s model to find  at 00:00 on 18th October 2017 | | | |
| **(c)** |  | | | |
| **B1ft:** | Either 8.5 or follow through “6 + their *R* ” (by using their *R* found in part (a)) | | | |
| **(d)** |  | | | |
| **M1:** | Realises that  and  so maximum depth occurs when | | | |
| **M1:** | Uses the model to deduce that a p.m. solution occurs when  and rearranges | | | |
|  | this equation to make | | | |
| **A1:** | Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm | | | |
| **(d)** |  | | | |
| **Alt 1** |  | | | |
| **M1:** | Maximum depth occurs when | | | |
| **M1:** | Rearranges to make  and adds on the period, where | | | |
| **A1:** | Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm | | | |

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| **Question 13 Notes Continued:** | |
| **(e)(i)** |  |
| **M1:** | See scheme |
| **A1:** | See scheme |
|  | **Note:** Allow Special Case M1 for a candidate who just states that Jolene’s model is not continuous at 00:00 on 19th October 2017 o.e. |
| **(e)(ii)** |  |
| **B1:** | Uses the information to set up a new model for *H*. (See scheme) |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
| **14** | |  |  |  |
|  | M1 | 3.1a |
| M1 | 1.1b |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
|  |  |  |
|  | A1 | 2.1 |
|  | **(5)** |  |
| **14**  **Alt 1** | |  |  |  |
|  | M1 | 3.1a |
| M1 | 1.1b |
|  | A1 | 1.1b |
| So, | M1 | 3.1a |
|  | A1 | 2.1 |
|  | **(5)** |  |
| **(5 marks)** | | | | |
| **Question 14 Notes:** | | | | |
| **M1:** | Looks ahead to the final result and uses the compound angle formula in a full attempt to write down an expression for  which is in terms of *t* only. | | | |
| **M1:** | Applies the compound angle formula on their term in *x*. E.g. | | | |
| **A1:** | Uses correct algebra to find | | | |
| **M1:** | Complete strategy of applying  on a rearranged ,  to achieve an equation in *x* and *y* only | | | |
| **A1:** | Correctly proves  with both , and no errors seen | | | |

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| **Question 14 Notes Continued:** | |
| **Alt 1** |  |
| **M1:** | Apply in the same way as in the main scheme |
| **M1:** | Apply in the same way as in the main scheme |
| **A1:** | Uses correct algebra to find  or |
| **M1:** | Complete strategy of applying  on  to achieve an equation in *x* and *y* only |
| **A1:** | Correctly proves  with both , and no errors seen |