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| **Pearson Edexcel Level 3** |
| **GCE Mathematics** **Advanced** **Paper 1: Pure Mathematics** |
| **Mock paper Spring 2018** **Time: 2 hours** | **Paper Reference(s)** |
| **9MA0/01** |
| **You must have:** **Mathematical Formulae and Statistical Tables, calculator** |

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

**Instructions**

• Use black ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

• Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions in the spaces provided – *there may be more space than you need*.

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Inexact answers should be given to three significant figures unless otherwise stated.

**Information**

• A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

• There are 14 questions in this paper. The total mark is 100.

• The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

**Advice**

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

 • If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**Answer ALL questions.**

**1.**

**Figure 1**

Figure 1 shows a sketch of the curve with equation *y* = **, *x* ≥ 0.

The finite region *R*, shown shaded in Figure 1, is bounded by the curve, the line with equation *x*= 1, the *x-*axis and the line with equation *x* = 3.

The table below shows corresponding values of *x* and *y* for *y* = **.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | 1 | 1.5 | 2 | 2.5 | 3 |
| *y* | 0.5 | 0.6742 | 0.8284 | 0.9686 | 1.0981 |

(*a*) Use the trapezium rule, with all the values of *y* in the table, to find an estimate for the area of *R*, giving your answer to 3 decimal places.

**(3)**

(*b*) Explain how the trapezium rule can be used to give a better approximation for the area of *R*.

**(1)**

(*c*) Giving your answer to 3 decimal places in each case, use your answer to part (a) to deduce an estimate for

 (i) , (ii) **.

**(2)**

 **(Total for Question 1 is 6 marks)**

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**2.** (*a*) Show that the binomial expansion of **in ascending powers of *x*, up to and including the term in *x*2 is

2 + *x* + *kx*2,

 giving the value of the constant *k* as a simplified fraction.

**(4)**

(*b*) (i) Use the expansion from part (a), with *x* = **, to find an approximate value for √2.

 Give your answer in the form **, where *p* and *q* are integers.

 (ii) Explain why substituting *x* = ** into this binomial expansion leads to a valid approximation.

**(4)**

**(Total for Question 2 is 8 marks)**

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**3.** A sequence of numbers *a*1, *a*2, *a*3 ,..., is defined by

*a*1 = 3,

*an* + 1 = **, *n* ∈ ℕ.

(*a*) Find **.

**(3)**

(*b*) Hence find ** + **

**(1)**

**(Total for Question 3 is 4 marks)**

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**4.** Relative to a fixed origin *O*,

 the point *A* has position vector **i** + 7**j** – 2**k**,

 the point *B* has position vector 4**i** + 3**j** + 3**k**,

 and the point *C* has position vector 2**i** + 10**j** + 9**k**.

Given that *ABCD* is a parallelogram,

(*a*) find the position vector of point *D*.

**(2)**

The vector ** has the same direction as **.

Given that |**| = 10√2,

(*b*) find the position vector of *X*.

**(3)**

**(Total for Question 4 is 5 marks)**

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**5.** f(*x*) = *x*3 + *ax*2 – *ax* + 48, where *a* is a constant.

Given that f(–6) = 0,

(*a*) (i) show that *a* = 4.

 (ii) express f(*x*) as a product of two algebraic factors.

**(4)**

Given that 2 log2 (*x* + 2) + log2 *x* – log2 (*x* – 6) = 3,

(*b*) show that *x*3 + 4*x*2 – 4*x* + 48 = 0.

**(4)**

(*c*) Hence explain why 2 log2 (*x* + 2) + log2 *x* – log2 (*x* – 6) = 3 has no real roots.

**(2)**

**(Total for Question 5 is 10 marks)**

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**6.**

**Figure 2 Figure 3**

Figure 2 shows the entrance to a road tunnel. The maximum height of the tunnel is measured as 5 metres and the width of the base of the tunnel is measured as 6 metres.

Figure 3 shows a quadratic curve *BCA* used to model this entrance.

The points *A*, *O*, *B* and *C* are assumed to lie in the same vertical plane and the ground *AOB* is assumed to be horizontal.

(*a*) Find an equation for curve *BCA*.

**(3)**

A coach has height 4.1 m and width 2.4 m.

(*b*) Determine whether or not it is possible for the coach to enter the tunnel.

**(2)**

(*c*) Suggest a reason why this model may not be suitable to determine whether or not the coach can pass through the tunnel.

**(1)**

 **(Total for Question 6 is 6 marks)**

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**7.**

**Figure 4**

Figure 4 shows a sketch of part of the curve with equation

*y* = 2e2*x* – *x*e2*x*, *x* ∈ ℝ.

The finite region *R*, shown shaded in Figure 4, is bounded by the curve, the *x-*axis and the *y‑*axis.

Use calculus to show that the exact area of *R* can be written in the form *p*e4 + *q*, where *p* and *q* are rational constants to be found.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(Total for Question 7 is 5 marks)**

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**8.** There were 2100 tonnes of wheat harvested on a farm during 2017.

The mass of wheat harvested during each subsequent year is expected to increase by 1.2% per year.

(*a*) Find the total mass of wheat expected to be harvested from 2017 to 2030 inclusive, giving your answer to 3 significant figures.

**(2)**

Each year it costs

* £5.15 per tonne to harvest the first 2000 tonnes of wheat
* £6.45 per tonne to harvest wheat in excess of 2000 tonnes

(*b*) Use this information to find the expected cost of harvesting the wheat from 2017 to 2030 inclusive. Give your answer to the nearest £1000.

**(3)**

 **(Total for Question 8 is 5 marks)**

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**9.** The curve *C* has equation *y* = 2*x*3 + 5.

The curve *C* passes through the point *P*(1, 7).

Use differentiation from first principles to find the value of the gradient of the tangent to *C* at *P*.

**(Total for Question 9 is 5 marks)**

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**10.** The function f is defined by

f : *x*  ↦ **, *x* ∈ ℝ, *x* ≠ –1.

(*a*) Find f –1(*x*).

**(3)**

(*b*) Show that

ff(*x*) = ** *x* ∈ ℝ, *x* ≠ –1,

 where *a* is an integer to be found.

**(4)**

The function g is defined by

g : *x* ↦ *x*2 – 3*x*, *x* ∈ ℝ, 0 ≤ *x* ≤ 5.

(*c*) Find the value of fg(2).

**(2)**

(*d*) Find the range of g.

**(3)**

(e) Explain why the function g does not have an inverse.

**(1)**

 **(Total for Question 10 is 13 marks)**

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**11.**

**Figure 5**

Figure 5 shows a sketch of the curve *C* with equation *y* = f(*x*).

The curve *C* crosses the *x-*axis at the origin, *O*, and at the points *A* and *B* as shown in Figure 5.

Given that f ′(*x*) = *k* – 4*x* – 3*x*2, where *k* is a constant,

(*a*) show that *C* has a point of inflection at *x* = –**.

**(3)**

Given also that the distance *AB* = 4√2,

(*b*) find, showing your working, the integer value of *k*.

**(7)**

 **(Total for Question 11 is 10 marks)**

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**12.** Show that

** d*θ* = 2 – 2 ln 2.

**(Total for Question 12 is 7 marks)**

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**13.** (*a*) Express 2 sin *θ* – 1.5 cos *θ* in the form *R* sin (*θ* – *α*), where *R* > 0 and 0 < *α* < **.

 State the value of *R* and give the value of *α* to 4 decimal places.

**(3)**

Tom models the depth of water, *D* metres, at Southview harbour on 18th October 2017 by the formula

*D* = 6 + 2 sin ** – 1.5 cos **, 0 ≤ *t* ≤ 24,

where *t* is the time, in hours, after 00:00 hours on 18th October 2017.

Use Tom’s model to

(*b*) find the depth of water at 00:00 hours on 18th October 2017,

**(1)**

(*c*) find the maximum depth of water,

**(1)**

(*d*) find the time, in the afternoon, when the maximum depth of water occurs.

 Give your answer to the nearest minute.

**(3)**

Tom’s model is supported by measurements of *D* taken at regular intervals on 18th October 2017. Jolene attempts to use a similar model in order to model the depth of water at Southview harbour on 19th October 2017.

Jolene models the depth of water, *H* metres, at Southview harbour on 19th October 2017 by the formula

*H* = 6 + 2 sin ** – 1.5 cos **, 0 ≤ *x* ≤ 24,

where *x* is the time, in hours, after 00:00 hours on 19th October 2017.

By considering the depth of water at 00:00 hours on 19th October 2017 for both models,

(*e*) (i) explain why Jolene’s model is not correct,

 (ii) hence find a suitable model for *H* in terms of *x*.

**(3)**

 **(Total for Question 13 is 11 marks)**

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**14*.***

**Figure 6**

Figure 6 shows a sketch of the curve *C* with parametric equations

*x* = 4 cos **, *y* = 2 sin *t*, 0 < *t* ≤ 2*π*.

Show that a Cartesian equation of *C* can be written in the form

(*x* + *y*)2 + *ay*2 = *b*,

where *a* and *b* are integers to be found.

**(5)**

 **(Total for Question 14 is 5 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**