

## Section 2: Mean values and general integration

### Exercise level 2

- The mean value of  $y = x^2$  in the interval  $[0, a]$  is equal to the mean value of  $y = x^3$  in the interval  $[0, a]$ . Find the value of  $a$ .
- Which of these three integrals is the biggest? Evaluate each one to find out.
  - $\int_1^{24} \frac{4}{\sqrt{9+16x^2}} dx$
  - $\int_1^2 \frac{9}{\sqrt{16-4x^2}} dx$
  - $\int_1^2 \frac{16}{\sqrt{9x^2-4}} dx$
- Find the exact value of  $\int_0^2 \frac{1}{3x^2+4} dx$ .
  - Use the substitution  $x\sqrt{3} = 2 \tan \theta$  to show that  $\int_0^2 \frac{1}{(3x^2+4)^{\frac{3}{2}}} dx = \frac{1}{8}$ .
- Use a suitable trigonometric or hyperbolic substitution to find:
  - $\int \sqrt{1-4x^2} dx$
  - $\int \sqrt{x^2-9} dx$
  - $\int_0^2 \frac{1}{(4+x^2)^{\frac{3}{2}}} dx$
  - $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{1}{(4-9x^2)^{\frac{3}{2}}} dx$
  - $\int_{0.5}^1 \frac{x^3}{\sqrt{4x^2-1}} dx$