Section 1: Further volumes of revolution

Exercise level 2

1. Find the volume of revolution produced when the curve $y = e^x$ between $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. Give your answer exactly in terms of π and e.

2. The section of the curve $y = \ln(1 + x)$ between $x = 0$ and $x = 1$ is rotated through 360° about the y-axis. Show that the volume of revolution V created is given by
   $\pi\int_0^{\ln 2}(e^{2y} - 2e^y + 1)\,dy$.
   Show that $V = \pi(\ln 2 - \frac{1}{2})$.

3. A curve is defined parametrically by the equations $x = t + t^2$, $y = t - t^2$.
   The region enclosed by the curve and the x-axis is rotated through 360° about the x-axis. Find the volume of the solid generated.

4. (i) Sketch the curve $y = 4\sinh x + 3\cosh x$.
   (ii) Write $(4\sinh x + 3\cosh x)^2$ in terms of $\sinh 2x$ and $\cosh 2x$.
   (iii) Find the volume of revolution formed by rotating the area enclosed by the axes and the curve in (i), through 360° about the x-axis.