**GCE AS Further Mathematics (8FM0) – Paper 23**

**Further Statistics 1**

**Summer 2019 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**This document is intended for guidance only and may differ significantly from the final mark scheme published in July 2019.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 6 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | H0: There is no association between age and activity  H1: There is an association between age and activity | B1 | This mark is given for both hypotheses in terms of “association” or independence” and mentioning both “age” and “activity” |
| (b) | =  = 6.24 | B1 | This mark is given for finding the correct expected frequency |
| (c) | Since expected value for *a* < 20 and snooker is  = 3.6 < 5,  two rows are amalgamated to give a 4 × 3 table | B1 | This mark is given for a reason for a 4 × 3 table from amalgamation of rows |
| The number of degrees of freedom is thus (4 – 1) × (3 – 1) = 6 | B1 | This mark is given for showing 6 degrees of freedom come from a 4 ×3 table |
| (d) | Critical value (5%) = 12.592 | B1 | This mark is given for a correct critical value |
| The result is significant, suggesting there is evidence to support the manager’s belief | B1 | This mark is given for a correct comparison and conclusion |

**Question 2 (Total 7 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | H0: Spinner is giving scores as it was designed to do  H1: Spinner is not giving scores as it was designed to do | B1 | This mark is given for both hypotheses in a suitable context |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | *Ei* | 24 | 8 | 8 | 32 | 8 | | *Oi* | 15 | 4 | 12 | 41 | 8 | |  |  |  |  |  | 0 | | M1 | This mark is given for using the model to find at least two correct expected frequencies |
| A1 | This mark is given for all expected frequencies (*Ei*) corrcet |
| = 3.375 + 2 + 2 + 2.53125 + 0 | M1 | This mark is given for a method to find a suitable test statistic (at least two terms correct) |
| = 9.90625 | A1 | This mark is given for finding the correct test statistic |
| *ν* = 5 – 1 = 4 so  (10%) critical value = 7.779 | B1 | This mark is given for finding the correct critical value |
| The result is significant, so there is evidence that the spinner is not giving scores as it is designed to do | A1 | This mark is given for a correct conclusion in context |

**Question 3 (Total 13 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | Let *X* = the number of errors in a 100 word piece  *X* ~ Po(1.7) | M1 | This mark is given for selecting the correct Poisson distribution |
| P(*X* < 2) = P(*X* ≥ 1) = 0.493… | A1 | This mark is given for finding the correct probability |
| (b) | Let *R* = the number of errors in the article  *R* ~ Po(4.25) | M1 | This mark is given for selecting the correct Poisson distribution |
| P(*R* = 5) = 0.165 | A1 | This mark is given for finding the correct probability |
| (c) | Scheme **A**: Let *A* ~ B(40, e–1.7) | M1 | This mark is given for choosing a correct model for Scheme **A** |
| P(*A* > 10) = 1 – P(*A* ≤ 10) | M1 | This mark is given for a method to find the probability of a bonus |
| = 0.0996 | A1 | This mark is given for finding the correct probability of a bonus |
| Scheme **B**: Let *B* ~ Po(40 × 1.7 ) | M1 | This mark is given for choosing a correct Poisson model for Scheme **A** |
| P(*B* < 56) = P(*B* ≤ 55) = 0.0611  Choose Scheme A since the probability of a bonus is greater (0.0996 > 0.611) | A1 | This mark is given for a correct conclusion based on a comparison of two probabilities |
| (d) | H0: *λ* = 1.7 (or *µ* = 8.5)  H1: *λ* < 1.7 (or *µ* < 8.5) | B1 | This mark is given for both hypotheses in terms of *λ* or *µ* |
| Let *E* = the number of errors in the piece of work  *E* ~ Po(8.5) | M1 | This mark is given for choosing a correct Poisson model |
| P(*E* ≤ 3) = 0.0301 or P(*E* ≤ 4) = 0.0744 | A1 | This mark is given for finding probabilities using Po(8.5) |
| Critical region in *E* ≤ 3 | A1 | This mark is given for finding a correct critical region |

**Question 4 (Total 14 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** | |
| (a) | *q* + | B1 | This mark is given for finding a correct value for P(*X* ≤ 0) | |
| (b) | E(*X* 2) =  (–3)2 × *q* + (–1)2 ×  + 12 ×  + 22 × *q* + 42 × *r* | | M1 | This mark is given for a method to find an expression for E(*X* 2) (at least three terms correct) |
| =  + 13*q* + 16*r* | A1 | This mark is given for a fully correct working leading to a correct answer | |
| (c) | E(*X*) = –3*q* –  +  + 2*q* + 4*r*  *=* 4*r* – *q* | M1 | This mark is given for recognising the need to find E(*X*) with at least three correct terms | |
| E(*X* 2 + 6*X*) =  + 7*q* + 40*r* | A1 | This mark is given for a correct expression for E(*X* 2 + 6*X*) | |
| E(*X* 3) =  (–3)3 × *q* + (–1)3 ×  + 13 ×  + 23 × *q* + 43 × *r* | | M1 | This mark is given for a method to find E(*X* 3) with at least three correct terms |
| = 64*r* – 19*q* | A1 | This mark is given for a correct expression for E(*X* 3) | |
| 2*q* + *r* = | M1 | This mark is given for using the sum of probabilities adding to 1 to form an equation in *q* and *r* | |
| 24*r* – 26*q* = ,  *r* + 2*q* = , 37*r* = | M1 | This mark is given for solving two linear equations in *q* and *r* | |
| *r* =  and *q* = | A1 | This mark is given for the two correct values of *q* and *r* | |
| (d) | *X* 3 > *X* 2 + 6*X* ⇒ *X*(*X* – 3)(*X* + 2) > 0 | M1 | This mark is given for rearranging and factorising the quadratic inequality | |
| From table, –2 < *X* < 0 or *X* > 3 | A1 | This mark is given for solving the inequality formed | |
| P(*X* 3 > *X* 2 + 6*X*) = P(*X* = ­1 or 4) | M1 | This mark is given for identifying the values of *X* required | |
| =  +  = | A1 | This mark is given for the correct probability | |