**GCE AS Further Mathematics (8FM0) – Paper 1**

**Core Pure Mathematics**

**Summer 2019 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**This document is intended for guidance only and may differ significantly from the final mark scheme published in July 2019.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 6 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | det **M** = (4 × –7) – (2 × –5) = –18 | M1 | This mark is given for a method to find the determinant of **M** |
| **M** is non-singular since det **M** ≠ 0 | A1 | This mark is given for a correct reason and an understanding that a non‑singular matrix has a non-zero determinant. |
| (b) | ⎜det **M** ⎜× Area *R* = Area *S* | M1 | This mark is given for recognising the relationship between the size of det **M** and the areas of triangles *R* and *S* |
| Area *R* =  = | A1 | This mark is given for finding the area of triangle *R* |
| (c) | = | M1 | This mark is given for mapping the line *y* = 2*x* under the transformation *T* |
| =  = –6  All points of *y* = 2*x* map to points on *y* = 2*x*, hence the line *y* = 2*x* is invariant | A1 | This mark is given correct multiplication and working leading to the conclusion that the line is invariant under the transformation *T* |

**Question 2 (Total 5 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** | | |
|  | *w* = *x* + 3, so *x* = *w* – 3 | B1 | This mark is given for finding the relationship between *x* and *w* | | |
| 2(*w* – 3)3 + 6(*w* – 3)2 – 3(*w* – 3) + 12 = 0 | M1 | This mark is given for substituting *w* – 3 into the cubic equation | | |
| 2*w*3 – 18*w*2 + 54*w* + 6*w*2 – 36*w* + 54 – 3*w* + 9 + 12 = 0 | | | M1 | This mark is given for multiplying out all the terms correctly |
| 2*w*3 – 12*w*2 + 15*w* + 21 = 0  (so *p* = 2, *q* = –12, *r* = 15 and *s* = 21) | A1 | This mark is given for at least two of *p*, *q*, *r* and *s* correct | | |
| A1 | This mark is given for all four terms *p*, *q*, *r* and *s* correct | | |

**Question 3 (Total 6 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | When *n* = 1,  =  =  and  =  = | B1 | This mark is given for checking both sides of the statement for *n* = 1 |
| Assume the statement is true for *n* = *k*, then  =  is true | M1 | This mark is given for assuming the general statement *n* = *k* is true |
| =  +  =  + | M1 | This mark is given for adding the (*k* + 1)th term to the sum of *k* terms |
| = | M1 | This mark is given for combining the two fractions over a common denominator |
| =  =  =  = | A1 | This mark is given for correct algebra leading to the term |
| Thus the general result is true for *n* = *k* + 1  Since the general result is true for *n* = 1 and true for *n* = *k* implies true for *n* = *k* + 1, the result is true for all *n* ∈ℕ | A1 | This mark is given for a fully correct induction statement |

**Question 4 (Total 5 marks)**

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| --- | --- | --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
|  | **r =** | M1 | | This mark is given for finding a parametric form for the line *l* | |
| The line *l* and the plane *Π* meet if  **.** = –7 | M1 | | This mark is given for substituting into the equation of the plane to find any points of intersection | |
| ⇒ (–2 + *λ*) × 1 + (5 –*λ*) × –2 + (4 – 3*λ*) × 1 = –7 | | A1 | | This mark is given for finding a correct equation in *λ* |
| ⇒ 0*λ* – 8 = –7  ⇒ –8 = –7, a contradiction | A1 | | This mark is given for simplifying the equation and deriving a contradiction | |
| Hence the line *l* is parallel to the plane *Π* but not in it, so there is no intersection | A1 | | This mark is given for a correct deduction following correct working | |

**Question 5 (Total 9 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | The complex roots of a real polynomial occur in conjugate pairs | M1 | This mark is given for a correct statement that complex roots of a real polynomial occur in conjugate pairs |
| A polynomial with roots *z*1, *z*2 and *z*3 also needs roots *z*2\* and *z*3\* (five in total)  However, a quartic has at most four roots, so no quartic can have *z*1, *z*2 and *z*3 roots. | A1 | This mark is given for s correct statement referencing that a quartic has four roots but would need five if *z*1, *z*2 and *z*3 were to all be roots. |
| (b) | =  =  × | M1 | This mark is given for substituting into the expression and multiplying the numerator and denominator by the conjugate of the denominator to find the quotient |
| =  =  =  + i | A1 | This mark is given for simplifying to find  + i |
| arctan  = arctan (1) and  + i is in the first quadrant;  hence arg  = | A1 | This mark is given for using arctan (1) and making reference to the first quadrant to justify the argument |
| (c) | arg  = arg (*z*2 – *z*1) – arg (*z*3 – *z*1)  = arg (–1 + 2i – (–2)) – arg (1 + i – (–2))  = arg (1 + 2i) – arg (3 + i) | M1 | This mark is given for applying the formula for the argument of a difference of complex numbers |
| Hence arctan (2) – arctan  = arg  = | A1 | This mark is given for a complete proof identifying the required arguments |
| (d) |  | B1 | This mark is given for a line passing through *z*2 with one side shaded |
| B1 | This mark is given for the area below and left of the line shaded |

**Question 6 (Total 9 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
| (a) | = | M1 | | This mark is given for finding a correct expression for the mean mass of the marbles, | |
| = 7 + 3  = 7*n* + 3(*n* + 1) | M1 | | This mark is given for correctly splitting the sum and using the arithmetic series formula | |
| = 7 + 3(*n* + 1)  =  = (3*n* + 17) | A1 | | This mark is given for correct working to arrive at the answer shown | |
| (b) | Standard deviation = | B1 | | This mark is given for using the formula for standard deviation (given in the formulae booklet) | |
| When *n* = 85, = (3 × 85 + 17) = 136 | B1 | | This mark is given for finding the mean mass of the marbles when *n* = 85 | |
| =  = | M1 | | This mark is given for a method to find an expression for | |
| = 49*n* + 42 × *n*(*n* + 1) + 9 × *n*(*n* + 1)(2*n* + 1) | | B1 | | This mark is given for an expression for  in terms of *n* |
| When *n* = 85,  =  = √5418 | M1 | | This mark is given for substituting to find a value for the standard deviation for the mass of the marbles when *n* = 85 | |
| = 73.6 g | A1 | | This mark is given for a value of the standard deviation of the mass of the marbles (accept any answer which rounds to 74, units not needed) | |

**Question 7 (Total 8 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *α* + *β* +  = 8 | M1 | This mark is given for finding the sum of roots and equating them to 8 |
| 2*α* +  = 8 | A1 | This mark is given for finding an equation in terms of *α* only. |
| 2*α* 2 + 12 = 8*α* so 2*α* 2 – 8*α* + 12 = 0  *α* = | M1 | This mark is given for rearranging to find a quadratic equation to solve |
| *α* = 2 ± i√2 | A1 | This mark is given for finding the two complex roots |
| Product of roots = 24  Third root =  = | M1 | This mark is given for a method to find the third root of f(*z*) |
| Hence the roots of f(*z*) are 2 + i√2, 2 – i√2 and 4 | A1 | This mark is given for finding the three roots of f(*z*) |
| (b) | f(4) = 0 ⇒ 43 – (8 × 42) + 4*p* – 24 = 0  4*p* = 88 | M1 | This mark is given for a method to find a value of *p* |
| *p* = 22 | A1 | This mark is given for a correct value for *p* |

**Question 8 (Total 12 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
| (a) | **b** = ± (*OQ* – *OP*)  = ±  = ± | M1 | | This mark is given for finding the positions between *P* and *Q* | |
| **r** =  + *λ* | A1 | | This mark is given for a correct vector equation in the form **r** = **a** + *λ***b** | |
| (b)(i) | 200 + 3*k* – 500 = 300  *k* = 200 | B1 | | This mark is given for substituting the coordinates of *M* into the equation of the plane modelling the side of the mountain and deducing a correct value of *k* | |
| If *M* is the point on the mountain and *X* a general point on the line, then  =  + *λ* –  = | M1 | | This mark is given for finding the distance from the point *M* on the mountain to a general point on the line | |
| **.** = 0 ⇒ *λ* = | M1 | | This mark is given for taking the dot product with the direction vector of the line and equating to zero to find a value of *λ* | |
| = + | M1 | | This mark is given for substituting *λ* = | |
|  | A1 | | This mark is given for finding the correct coordinates of the point at which the tunnel meets the pipeline | |
| (b)(ii) |  | | M1 | | This mark is given for a method to find the length of the tunnel |
| = √48750 ≈ 221 m | A1 | | This mark is given for correct length of the tunnel (including units) | |

**Question 8 (continued) (Total 12 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (c) | = ≈ 522  = ≈ 427 | M1 | This mark is given for calculating the two lengths of the existing accessways *OP* and *OQ* |
| The new accessway length is considerably shorter than 522 m and 427 m, so the company should build the new accessway | A1 | This mark is given for an appropriate conclusion |
| (d) | For example:  The mountainside in not likely to be flat, so a plane might not be a good model  The tunnel and pipelines will not have negligible thickness so modelling as mines might not be appropriate  The shortest length for a tunnel might not be practicable depending on the type of rock in the mountain | B1 | This mark is given for a valid limitation of the model stated |

**Question 9 (Total 8 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
|  |  | M1 | This mark is given for an attempt at integrating *y*2 with respect to *x* in volume of revolution formulae. |
| *y*2 = ( + ) ( + )  =  +  + | M1 | This mark is given for ,  and  seen, regardless of the coefficients |
| A1 | This mark is given for a fully correct expression for *y*2 |
| =  =  +  – | M1 | This mark is given for two of ,  and  seen, regardless of coefficients |
| A1 | This mark is given for two terms of the integral given correctly |
| A1 | This mark is given for a fully correct integral |
| =  ⇒ (99.3 + 4.425) = | M1 | This mark is given for enumerating the integral to solve for *θ* |
| ⇒ *θ* = 2 ×  = 4.444…  *θ* =  (radians) | A1 | This mark is given for the correct angle found |

**Question 10 (Total 12 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
| --- | --- | --- | --- | --- | --- |
| (a) | *a* represents the proportion of juvenile chimpanzees that survive and remain juvenile chimpanzees next year | B1 | | This mark is given for a correct interpretation of *a* | |
| (b)(i) | Determinant = 0.82*a* – (0.08 × 0.15) | M1 | | This mark is given for finding the determinant of the matrix | |
| = … | M1 | | This mark is given for a method to find the inverse, with swapped leading diagonals and signs changed on the off diagonals | |
| = | A1 | | This mark is given for a fully correct inverse matrix | |
| (b)(ii) | =  = | M1 | | This mark is given for using the inverse matrix to find the initial juvenile and adult populations of chimpanzees | |
| [6144 + (43008*a* – 1228.8) = 64 000  4915.2 + 43008*a* = 64 000(0.82*a* – 0.012) | | M1 | | This mark is given for using the sum of initial populations adding up to 64000 to find the value of *a* |
| *a* =  = 0.60 | A1 | | This mark is given for finding the correct value of *a* | |
| (b)(iii) | Initial juvenile population  =  =  = 12800 | M1 | | This mark is given for using the value of *a* to find *J*0 | |
| 15 360 – 12 800 = 2560 | A1 | | This mark is given for finding the change (increase) in the number of juvenile chimpanzees in the first year of the study | |
| (c) | Since the number of juvenile chimpanzees has increased, the model is not initially predicting a decline so is not suitable in the short term | B1 | | This mark is given for a valid comment | |

**Question 10 continued (Total 12 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (d) | A third category needs to be introduced for chimpanzees aged 40 and above and for which there is no birth rate (no new juveniles produced) | M1 | This mark is given for identifying a third category to be added to the model |
| Matrix model with form  =  since mature chimpanzees (*Mn*) cannot become juvenile or adult, and juveniles cannot become mature in one year | A1 | This mark is given for a matrix model in the correct form with a valid explanation |