Topic assessment

1. Find the volume of the solid generated when the region bounded by the curve $y = 3x - x^2$, the x axis and the lines $x = 1$ and $x = 2$, is rotated through 360° about the x axis. [5]

2. Find the volume formed by rotating completely about the x axis the region bounded by the curve $y = x^3 - 2x^2$ and the x axis. [5]

3. The region formed by $y = \sqrt{x}$, $y = 2$ and the y axis is rotated through 360° about the y axis. Find the volume generated. [5]

4. A curve has equation $y = \dfrac{1}{(1+x)^3}$.

   (i) Sketch the curve for $x > 0$ and shade the region enclosed by the coordinate axes and the line $x = 1$. [3]

   (ii) The shaded region is rotated through 360° about the y-axis. Find the volume of the solid generated.

   [7]

Total 25 marks

Solutions to topic assessment

1. $\text{Volume} = \int_1^2 \pi y^2 dx$

$$= \pi \int_1^2 (3x - x^2)^2 dx$$
$$= \pi \int_1^2 (9x^2 - 6x^3 + x^4) dx$$
$$= \pi \left[3x^3 - \tfrac{3}{2}x^4 + \tfrac{1}{5}x^5 \right]_1^2$$
$$= \pi \left(24 - 24 + \tfrac{32}{5} - (3 - \tfrac{3}{2} + \tfrac{1}{5}) \right)$$
$$= \tfrac{47}{10}\pi$$

[5]

2. $y = x^3 - 2x^2$

When $y = 0$, $x^2(x-2) = 0$

$x = 0$ or $x = 2$

$\text{Volume} = \int_0^2 \pi y^2 dx$

$$= \pi \int_0^2 (x^3 - 2x^2)^2 dx$$
$$= \pi \int_0^2 (x^6 - 4x^5 + 4x^4) dx$$
$$= \pi \left[\tfrac{1}{7}x^7 - \tfrac{2}{3}x^6 + \tfrac{4}{5}x^5 \right]_0^2$$
$$= \pi \left(\tfrac{128}{7} - \tfrac{128}{3} + \tfrac{128}{5} \right)$$
$$= \tfrac{128}{105}\pi$$

[5]

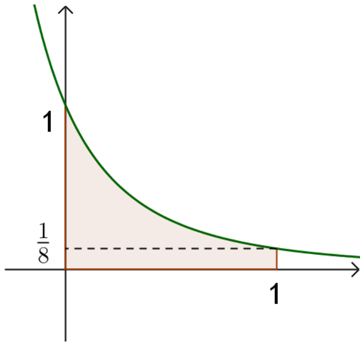
3. $y = \sqrt{x} \quad \Rightarrow x = y^2$

$\text{Volume} = \int_0^2 \pi x^2 dy$

$$= \pi \int_0^2 (y^2)^2 dy$$
$$= \pi \int_0^2 y^4 dy$$
$$= \pi \left[\tfrac{1}{5}y^5 \right]_0^2$$
$$= \pi \left(\tfrac{32}{5} - 0 \right)$$
$$= \tfrac{32}{5}\pi$$

[5]

4. (i)



[3]

(ii) $y = \frac{1}{(x+1)^3} \Rightarrow (x+1)^3 = \frac{1}{y} \Rightarrow x+1 = y^{-\frac{1}{3}} \Rightarrow x = y^{-\frac{1}{3}} - 1$

The volume of the part above the dotted line is given by

$$\begin{aligned}
\text{Volume } &= \int_{1/8}^{1} \pi x^2 dy \\
&= \int_{1/8}^{1} \pi (y^{-\frac{1}{3}} - 1)^2 dy \\
&= \int_{1/8}^{1} \pi (y^{-\frac{2}{3}} - 2y^{-\frac{1}{3}} + 1) dy \\
&= \pi \left[3y^{\frac{1}{3}} - 3y^{\frac{2}{3}} + y \right]_{1/8}^{1} \\
&= \pi \left(3 - 3 + 1 - (3 \times \tfrac{1}{2} - 3 \times \tfrac{1}{4} + \tfrac{1}{8}) \right) \\
&= \tfrac{1}{8}\pi
\end{aligned}$$

The volume of the part below the dotted line is a cylinder, radius 1 and height $\frac{1}{8}$, so this has volume $\pi \times 1^2 \times \frac{1}{8} = \frac{1}{8}\pi$

Total volume $= \frac{1}{8}\pi + \frac{1}{8}\pi = \frac{1}{4}\pi$

[7]