Topic assessment

1. Describe each of the following transformations.

(i) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (ii) $\begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$ (iii) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

[8]

2. Given that $\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 0 \end{pmatrix}$, find **AB** and **BA**. [4]

3. The matrix $\begin{pmatrix} -0.28 & 0.96 \\ -0.96 & -0.28 \end{pmatrix}$ corresponds to a rotation R in the x-y plane.

(i) State the centre of the rotation R, and find the angle of rotation (stating whether it is clockwise or anticlockwise). [3]

The transformation S is the rotation R followed by reflection in the x-axis.

(ii) Write down the matrix corresponding to reflection in the x-axis, and show that the matrix corresponding to the transformation S is $\begin{pmatrix} -0.28 & 0.96 \\ 0.96 & 0.28 \end{pmatrix}$. [3]

(iii) Show that the invariant points of the transformation S lie on a straight line, and find the equation of this line. [4]

4. The matrix $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ represents a transformation, T.

(i) Find the invariant points for the transformation T. [3]

(ii) T is a transformation called a shear. The *line of shear* is the line of invariant points for the shear. The *factor* of a shear gives the distance a point is moved as a multiple of its perpendicular distance from the line of shear. What is the factor of the shear T? [4]

5. The matrix $\begin{pmatrix} 3 & -2 \\ 4 & -6 \end{pmatrix}$ defines a transformation **M** of the (x, y) plane.

(i) Show that the origin is the only invariant point of the transformation. [2]

(ii) Find the two values of m for which $y = mx$ is an invariant line under the transformation. [4]

Total 35 marks

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Solutions to topic assessment

1. (i) Reflection in the x-axis [2]

   (ii) Rotation through 60° anticlockwise about the origin [2]

   (iii) Reflection in the line $y = -x$ [2]

   (iv) Rotation through 90° about the x-axis [2]

2. $$\mathbf{AB} = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ -9 & 7 \end{pmatrix}$$

   $$\mathbf{BA} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 3 \\ 1 & 8 & -9 \\ 8 & 4 & 0 \end{pmatrix}$$

   [4]

3. (i) The centre of rotation is the origin.

   Comparing with the general rotation matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ for an anticlockwise rotation of θ about the origin gives

   $$\cos\theta = -0.28$$

   $$\sin\theta = -0.96$$

   So θ is in the third quadrant, and therefore $\theta = -106.3°$ (1 d.p.)

   The angle of rotation is 106.3° clockwise.

   [3]

   (ii) For reflection in the x-axis, the point (1, 0) is mapped to itself, and the point (0, 1) is mapped to the point (0, -1)

   So the matrix corresponding to reflection in the x-axis is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

   $$\mathbf{S} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -0.28 & 0.96 \\ -0.96 & -0.28 \end{pmatrix} = \begin{pmatrix} -0.28 & 0.96 \\ 0.96 & 0.28 \end{pmatrix}$$

   [3]

(iii) For invariant points, $\begin{pmatrix} -0.28 & 0.96 \\ 0.96 & 0.28 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{pmatrix} -0.28x + 0.96y \\ 0.96x + 0.28y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -1.28x + 0.96y \\ 0.96x - 0.72y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Both these equations simplify to $4x = 3y$
so the invariant points all satisfy the equation $4x = 3y$, and so they all line on a straight line with equation $4x = 3y$.

[4]

4. (i) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

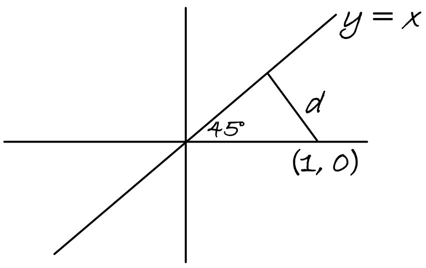
$$\begin{pmatrix} 3x - 2y \\ 2x - y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2x - 2y \\ 2x - 2y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The invariant points are all points on the line $y = x$

[3]

(ii) Consider the point (1, 0).
Under T the point (1, 0) is mapped to the point (3, 2), so the distance it moves is $\sqrt{2^2 + 2^2} = 2\sqrt{2}$.



The perpendicular distance of the point (1, 0) from the line of shear $y = x$ is given by $d = 1 \times \sin 45^\circ = \frac{1}{\sqrt{2}}$.

Factor of shear $= \frac{2\sqrt{2}}{1/\sqrt{2}} = 4$.

[4]

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5. (i) At invariant points, $\begin{pmatrix} 3 & -2 \\ 4 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{pmatrix} 3x-2y \\ 4x-6y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2x-2y \\ 4x-7y \end{pmatrix} = 0$$

This can only be true if $x = y = 0$, so the origin is the only invariant point.

[2]

(ii) For a point on the line $y = mx$, image is $\begin{pmatrix} 3 & -2 \\ 4 & -6 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} 3x-2mx \\ 4x-6mx \end{pmatrix}$

For an invariant line, the image point must lie on the line

so $4x-6mx = m(3x-2mx)$

$$4-6m = 3m-2m^2$$

$$2m^2 - 9m + 4 = 0$$

$$(2m-1)(m-4) = 0$$

$$m = \tfrac{1}{2} \quad \text{or} \quad m = 4$$

[4]