Topic assessment

1. The matrix $\begin{pmatrix} 3 & -2 \\ 4 & -6 \end{pmatrix}$ defines a transformation **M** of the (x, y) plane.

   A triangle S has area 3 square units, and **M** transforms S to a triangle T.

   (i) Find the area of T. [2]

   (ii) Find the matrix which transforms T to S. [2]

   (iii) Find the point which is mapped to the point (9, 2) [2]

2. The matrix $\begin{pmatrix} 1 & 2 & 3 \\ 3 & -1 & k \\ 1 & 0 & 1 \end{pmatrix}$ maps the unit cube to a solid with volume 4 cm^3.

   (i) Find the two possible values of k. [4]

   (ii) In the case for which the orientation of the image is unchanged from the orientation of the original cube, find the coordinates of the point P which is mapped to the point (0, 1, 2). [3]

3. The matrix $\mathbf{M} = \begin{pmatrix} 2 & -3 \\ a & 6 \end{pmatrix}$ is singular.

   (i) Find the value of a. [2]

   (ii) Show that **M** maps every point on the plane to a point on a straight line, and find the equation of this line. [3]

4. (i) Find the determinant of the matrix $\mathbf{M} = \begin{pmatrix} k & 2 & 3 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$ in terms of k. [3]

   (ii) State the value of k for which **M** is singular. [1]

   (iii) Given that **M** is non-singular, find $\mathbf{M}^{-1}$. [4]

5. You are given the matrix equation $\begin{pmatrix} 3 & -2 & -18 \\ 2 & 1 & -5 \\ 7 & k & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 25 \\ 20 \end{pmatrix}$.

   (i) Solve the equation when $k = -32$. [4]

   (ii) Show that if $k = 10$ the equation does not have a unique solution. Determine whether there is no solution or whether there are infinitely many solutions. Give a geometrical interpretation. [5]

6. Show that the equation $\begin{pmatrix} 3 & -7 & 0 \\ 2 & 2 & 5 \\ 1 & 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ does not have a unique solution, and give a geometrical interpretation. [5]

7. For the equations

$$3x - 3y - z = a$$
$$2x - y - z = 5$$
$$x + ky - 2z = 7$$

(i) Show that the equations do not have a unique solution if $k = 4$. [2]

(ii) Solve the equations for the case $k = 2$ and $a = 8$. [4]

(iii) In the case $k = 4$, find the value of a for which the equations are consistent. [4]

Total 50 marks

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Solutions to topic assessment

1. (i) $\det\begin{pmatrix}3 & -2\\ 4 & -6\end{pmatrix} = (3\times -6)-(4\times -2) = -18+8 = -10$

Area scale factor = 10, so area of T $= 3\times 10 = 30$ square units.

[2]

(ii) Inverse matrix $= \frac{1}{-10}\begin{pmatrix}-6 & 2\\ -4 & 3\end{pmatrix} = \begin{pmatrix}0.6 & -0.2\\ 0.4 & -0.3\end{pmatrix}$

[2]

(iii) $\begin{pmatrix}0.6 & -0.2\\ 0.4 & -0.3\end{pmatrix}\begin{pmatrix}9\\ 2\end{pmatrix} = \begin{pmatrix}5\\ 3\end{pmatrix}$

so the point mapped to (9, 2) is (5, 3). [2]

2. (i) $\left|\begin{pmatrix}1 & 2 & 3\\ 3 & -1 & k\\ 1 & 0 & 1\end{pmatrix}\right| = 1(-1-0)-2(3-k)+3(0+1)$

$= -1-6+2k+3$

$= 2k-4$

Since the volume factor is 4, the determinant is 4 or -4.

$2k-4=4$	$2k-4=-4$
$2k=8$	$2k=0$
$k=4$	$k=0$

$k = 0$ or 4

[4]

(ii) If the orientation is unchanged, the determinant is positive so this is the case for which $k=4$.

$\begin{pmatrix}1 & 2 & 3\\ 3 & -1 & k\\ 1 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}0\\ 1\\ 2\end{pmatrix}$

Inverse matrix $= \frac{1}{4}\begin{pmatrix}-1 & -2 & 11\\ 1 & -2 & 5\\ 1 & 2 & -7\end{pmatrix}$

$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \frac{1}{4}\begin{pmatrix}-1 & -2 & 11\\ 1 & -2 & 5\\ 1 & 2 & -7\end{pmatrix}\begin{pmatrix}0\\ 1\\ 2\end{pmatrix} = \frac{1}{4}\begin{pmatrix}20\\ 8\\ -12\end{pmatrix} = \begin{pmatrix}5\\ 2\\ -3\end{pmatrix}$

So P = (5, 2, -3).

[3]

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3. (i) Determinant is zero, so $12+3a=0$

$$a=-4$$

(ii) $\begin{pmatrix}2 & -3\\ -4 & 6\end{pmatrix}\begin{pmatrix}p\\ q\end{pmatrix}=\begin{pmatrix}2p-3q\\ -4p+6q\end{pmatrix}=\begin{pmatrix}2p-3q\\ -2(2p-3q)\end{pmatrix}$

So every point is mapped to a point on the line $y=-2x$.

[3]

4. (i) $\left|\begin{pmatrix}k & 2 & 3\\ 3 & 2 & -1\\ 2 & 1 & -1\end{pmatrix}\right|=k(-2+1)-2(-3+2)+3(3-4)$

$$=-k+2-3$$
$$=-k-1$$

[3]

(ii) $k=-1$

[1]

(iii) Matrix of cofactors $=\begin{pmatrix}-1 & 1 & -1\\ 5 & -k-6 & -k+4\\ -8 & k+9 & 2k-6\end{pmatrix}$

Inverse matrix $=\dfrac{1}{-k-1}\begin{pmatrix}-1 & 5 & -8\\ 1 & -k-6 & k+9\\ -1 & -k+4 & 2k-6\end{pmatrix}$

$$=\frac{1}{k+1}\begin{pmatrix}1 & -5 & 8\\ -1 & k+6 & -k-9\\ 1 & k-4 & 6-2k\end{pmatrix}$$

[4]

5. (i) When k = -32, inverse matrix = $\dfrac{1}{882}\begin{pmatrix}-158 & 580 & 28\\ -39 & 132 & -21\\ -71 & 82 & 7\end{pmatrix}$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{882}\begin{pmatrix}-158 & 580 & 28\\ -39 & 132 & -21\\ -71 & 82 & 7\end{pmatrix}\begin{pmatrix}6\\ 25\\ 20\end{pmatrix}.$$

$$=\frac{1}{882}\begin{pmatrix}14112\\ 2646\\ 1764\end{pmatrix}=\begin{pmatrix}16\\ 3\\ 2\end{pmatrix}$$

$x=16$, $y=3$, $z=2$

[4]

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(ii) $\begin{pmatrix} 3 & -2 & -18 \\ 2 & 1 & -5 \\ 7 & 10 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 25 \\ 20 \end{pmatrix} \Rightarrow \begin{array}{ll} 3x-2y-18z=6 & ① \\ 2x+y-5z=25 & ② \\ 7x+10y+2z=20 & ③ \end{array}$

①+2×② ⇒ $7x - 28z = 56$ ⇒ $x - 4z = 8$

5×①+③ ⇒ $22x - 88z = 50$ ⇒ $x - 4z = \frac{50}{22}$

so the equations are inconsistent, and there is no solution.

The equations represent three planes which form a triangular prism.

[5]

6. $3x - 7y = 3$ ①

$2x + 2y + 5z = 2$ ②

$x + 3y + 4z = 1$ ③

4×②–5×③ ⇒ $3x - 7y = 3$

Comparing with ①, the equations are consistent and there are infinitely many solutions.

The equations represent a sheaf of planes.

[5]

7. $\begin{pmatrix} 3 & -3 & -1 \\ 2 & -1 & -1 \\ 1 & k & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ 5 \\ 7 \end{pmatrix}$

(i) If k = 4 the determinant is zero so the equations do not have a unique solution.

[2]

(ii) $\begin{pmatrix} 3 & -3 & -1 \\ 2 & -1 & -1 \\ 1 & 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ 7 \end{pmatrix}$

If k = 2, determinant = -2.

Inverse matrix $= -\frac{1}{2}\begin{pmatrix} 4 & -8 & 2 \\ 3 & -5 & 1 \\ 5 & -9 & 3 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 4 & -8 & 2 \\ 3 & -5 & 1 \\ 5 & -9 & 3 \end{pmatrix}\begin{pmatrix} 8 \\ 5 \\ 7 \end{pmatrix}$$

$$= -\frac{1}{2}\begin{pmatrix} 6 \\ 6 \\ 16 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \\ -8 \end{pmatrix}$$

The solution is $x = -3$, $y = -3$, $z = -8$.

[4]

(iii) $3x - 3y - z = a$ (1)

$2x - y - z = 5$ (2)

$x + 4y - 2z = 7$ (3)

(1) – (2): $x - 2y = a - 5$

2×(2) – (3): $3x - 6y = 3 \Rightarrow x - 2y = 1$

The equations are consistent if $a - 5 = 1 \Rightarrow a = 6$

[4]