Topic assessment

1. The complex number α is given by $\alpha = -2 + 5\text{i}$.
   (i) Write down the complex conjugate α^*. [1]
   (ii) Find $\dfrac{\alpha + \alpha^*}{\alpha}$ in the form $a + b\text{i}$. [3]

2. Find the two square roots of the complex number $5 - 12\text{i}$. [5]

3. Solve the equation $(2+3\text{i})z + 3 - \text{i} = \text{i}z + 1$. [4]

4. The complex number w is given by $w = 1 + 2\text{i}$.
   (i) Express w^2, w^3 and w^4 in the form $a + b\text{i}$. [5]
   (ii) Given that w is a root of the equation $z^4 + pz^3 + qz^2 - 6z + 65 = 0$, find the values of p and q. [5]

5. The roots of the equation $z^2 - 4z + 5 = 0$ are z_1 and z_2.
   (i) Show that $z_1 = 2 + \text{i}$ and find the other root, z_2. [3]
   (ii) Show that $\dfrac{1}{z_1} + \dfrac{1}{z_2} = \dfrac{4}{5}$. [3]
   (iii) Show also that $\text{Im}\,(z_1^2 + z_2^2) = 0$ and find $\text{Re}\,(z_1^2 - z_2^2)$. [4]
   (iv) Find the complex numbers z_1^2, and z_1^3. [4]
   (v) Plot the three complex numbers z_1, z_1^2, and z_1^3 on an Argand diagram. [3]

Total 40 marks

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Solutions to topic assessment

1. $\alpha = -2 + 5i$

   (i) $\alpha^* = -2 - 5i$

   [1]

   (ii) $\dfrac{\alpha + \alpha^*}{\alpha} = \dfrac{-2 + 5i - 2 - 5i}{-2 + 5i}$

   $= \dfrac{-4}{-2 + 5i}$

   $= \dfrac{-4(-2 - 5i)}{(-2 + 5i)(-2 - 5i)}$

   $= \dfrac{8 + 20i}{29} = \dfrac{8}{29} + \dfrac{20}{29}i$

   [3]

2. $(a + bi)^2 = 5 - 12i$

   $a^2 + 2abi - b^2 = 5 - 12i$

   Equating imaginary parts: $2ab = -12$

   $b = -\dfrac{6}{a}$

   Equating real parts: $a^2 - b^2 = 5$

   $a^2 - \dfrac{36}{a^2} = 5$

   $a^4 - 5a^2 - 36 = 0$

   $(a^2 - 9)(a^2 + 4) = 0$

   $a = \pm 3$

   When $a = 3, b = -2$

   When $a = -3, b = 2$

   The two square roots are $3 - 2i$ and $-3 + 2i$.

3. $(2 + 3i)z + 3 - i = iz + 1$

   $(2 + 3i)z - iz = 1 - 3 + i$

   $(2 + 2i)z = -2 + i$

   $z = \dfrac{-2 + i}{2(1 + i)} = \dfrac{(-2 + i)(1 - i)}{2(1 + i)(1 - i)}$

   $= \dfrac{-2 + 3i + 1}{2(1 + 1)}$

   $= -\frac{1}{4} + \frac{3}{4}i$

   [4]

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4. (i) $w = 1 + 2i$

$w^2 = (1+2i)(1+2i) = 1 + 4i - 4 = -3 + 4i$

$w^3 = (-3+4i)(1+2i) = -3 - 2i - 8 = -11 - 2i$

$w^4 = (-11-2i)(1+2i) = -11 - 24i + 4 = -7 - 24i$

[5]

(ii) $z^4 + pz^3 + qz^2 - 6z + 65 = 0$

$-7 - 24i + p(-11-2i) + q(-3+4i) - 6(1+2i) + 65 = 0$

Equating real parts: $-7 - 11p - 3q - 6 + 65 = 0$

$11p + 3q = 52$ ①

Equating imaginary parts: $-24 - 2p + 4q - 12 = 0$

$p - 2q = -18$ ②

$2\times$① $+ 3\times$②: $25p = 50 \Rightarrow p = 2, \quad q = 10$

[5]

5. (i) $z_1^2 - 4z + 5 = (2+i)^2 - 4(2+i) + 5$

$= 4 + 4i - 1 - 8 - 4i + 5$

$= 0$

The other root, $z_2 = z_1{*} = 2 - i$

[3]

(ii) $\dfrac{1}{z_1} + \dfrac{1}{z_2} = \dfrac{1}{2+i} + \dfrac{1}{2-i}$

$= \dfrac{2-i+2+i}{(2+i)(2-i)}$

$= \dfrac{4}{4+1}$

$= \dfrac{4}{5}$

[3]

(iii) $z_1^2 + z_2^2 = (2+i)^2 + (2-i)^2$

$= 4 + 4i - 1 + 4 - 4i - 1$

$= 6$

$\mathrm{Im}\left(z_1^2 + z_2^2\right) = 0$

$z_1^2 - z_2^2 = (2+i)^2 - (2-i)^2$

$= 4 + 4i - 1 - 4 + 4i + 1$

$= 8i$

$\mathrm{Re}\left(z_1^2 - z_2^2\right) = 0$

[4]

(iv) $z_1^2 = (2+i)^2$

$= 4 + 4i - 1$

$= 3 + 4i$

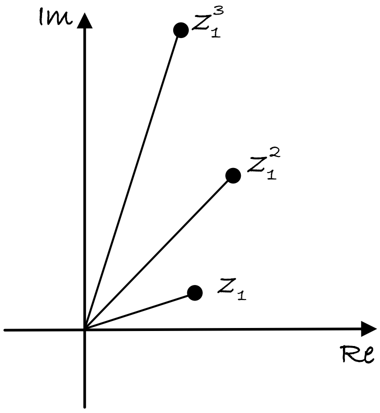
$z_1^3 = (3+4i)(2+i)$

$= 6 + 11i - 4$

$= 2 + 11i$

[4]

(v)



[3]