Topic assessment

1. A particle moves on the x-axis. Its displacement, x m, from the origin O is given by

$$x = 3t^2 - 3t + 2$$, where t is the time in seconds.

How far is the particle from O when it is instantaneously at rest? [5]

2. A racing car starts off down a straight section of track towards the first corner. Its speed, v ms^{-1}, is modelled for the first four seconds of its motion by

$$v = t^3 - 9t^2 + 24t, \qquad 0 \le t \le 4.$$

(i) Find an expression for the distance travelled by the car in the first t seconds.

Calculate the distance travelled from $t = 2$ to $t = 4$. [5]

(ii) Show that the acceleration, a ms^{-2}, of the car at time t is given by $a = k(t-2)(t-4)$, where k is a constant to be determined. [2]

3. The velocity, v, of a particle is given as

$$v = 2t^2 - 3t - \tfrac{1}{3}t^3.$$

(i) Show that the acceleration of the particle is zero when $t = 1$ and when $t = 3$. [3]

(ii) Calculate the displacement of the particle from its position when $t = 1$ to its position when $t = 2$. [4]

4. A car starts from rest and travels along a straight road. Its speed, v ms^{-1}, at time t seconds is modelled by

$$v = 4t - 0.2t^2, \qquad 0 \le t \le 10,$$
$$v = \text{constant}, \qquad 10 \le t \le 15,$$
$$v = 8 + 0.8t, \qquad t \ge 15.$$

(i) Calculate the speed of the car at $t = 0$, $t = 10$, $t = 15$ and $t = 20$. [3]

(ii) Find the values of the acceleration at
(A) $t = 7$,
(B) $t = 12$,
(C) $t = 16$. [4]

(iii) Calculate the distance the car travels in the interval $10 \le t \le 20$. [5]

(iv) Calculate the distance the car travels in the interval $0 \le t \le 10$. [4]

5. An insect moves in a straight line. The time, *t*, is in seconds and distance travelled is in metres.

   The velocity, v ms^{-1}, of the insect is given by

$$\begin{aligned} v &= t^2 - 4t, & 0 \le t \le 6, \\ v &= c, & 6 \le t \le 10, \\ v &= at + b, & 10 \le t \le 15. \end{aligned}$$

   You are also given that $v = 4$ when $t = 12$.

   (i) Show that $c = 12$. [2]

   (ii) Calculate the values of a and b and briefly describe the motion of the insect in the interval $10 \le t \le 15$. [4]

   (iii) Calculate the values of v for $t = 0$, $t = 2$ and $t = 4$. Sketch the v-t curve for the motion of the insect in the interval $0 \le t \le 6$. [3]

   (iv) Calculate the **distance** travelled by the insect in the interval $0 \le t \le 6$. [6]

Total 50 marks

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Solutions to topic assessment

1. $x = 3t^2 - 3t + 2$

$v = \dfrac{dx}{dt} = 6t - 3$

When particle is instantaneously at rest, $6t - 3 = 0$

$t = 0.5$

When t = 0.5, $x = 3 \times 0.5^2 - 3 \times 0.5 + 2$

$= 1.25$

It is 1.25 m from O when it is instantaneously at rest.

[5]

2. (i) $v = t^3 - 9t^2 + 24t$

$s = \int v\,dt = \frac{1}{4}t^4 - 3t^3 + 12t^2 + c$

When t = 0, s = 0 $\Rightarrow$ c = 0

$s = \frac{1}{4}t^4 - 3t^3 + 12t^2$

Distance travelled from t = 2 to t = 4 is

$\left(\frac{1}{4} \times 4^4 - 3 \times 4^3 + 12 \times 4^2\right) - \left(\frac{1}{4} \times 2^4 - 3 \times 2^3 + 12 \times 2^2\right)$

$= 64 - 192 + 192 - 4 + 24 - 48 = 36$

Distance travelled = 36 m.

[5]

(ii) $a = \dfrac{dv}{dt} = 3t^2 - 18t + 24 = 3\left(t^2 - 6t + 8\right) = 3(t - 2)(t - 4)$

so k = 3. [2]

3. (i) $v = 2t^2 - 3t - \frac{1}{3}t^3$

$a = \dfrac{dv}{dt} = 4t - 3 - t^2$

When a = 0, $t^2 - 4t + 3 = 0$

$(t - 1)(t - 3) = 0$

$t = 1$ or $t = 3$

[3]

(ii) Displacement $= \int_1^2 v\,dt$

$= \left[\frac{2}{3}t^3 - \frac{3}{2}t^2 - \frac{1}{12}t^4\right]_1^2$

$= \left(\frac{16}{3} - 6 - \frac{4}{3}\right) - \left(\frac{2}{3} - \frac{3}{2} - \frac{1}{12}\right)$

$= -\frac{13}{12}$

[4]

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4. $v = 4t - 0.2t^2$, $0 \le v \le 10$,

$v =$ constant, $10 \le t \le 15$,

$v = 8 + 0.8t$, $t \ge 15$.

(i) When t = 0, $v = 4 \times 0 - 0.2 \times 0^2 = 0$

When t = 10, $v = 4 \times 10 - 0.2 \times 10^2 = 20$

When t = 15, $v = 8 + 0.8 \times 15 = 20$

When t = 20, $v = 8 + 0.8 \times 20 = 24$

[3]

(ii) (A) When t = 7, $v = 4t - 0.2t^2$

$$a = \frac{dv}{dt} = 4 - 0.4t$$

Acceleration $= 4 - 0.4 \times 7 = 1.2\ ms^{-2}$

(B) When t = 12, $v = 20$

$$a = \frac{dv}{dt} = 0$$

Acceleration = 0

(C) When t = 16, $v = 8 + 0.8t$

$$a = \frac{dv}{dt} = 0.8$$

Acceleration $= 0.8\ ms^{-2}$.

[4]

(iii) In the interval $10 \le t \le 15$, speed is constant.

Distance travelled $= 20 \times 5 = 100$

In the interval $15 \le t \le 20$, acceleration is constant.

$u = 20$ $\quad s = ut + \frac{1}{2}at^2$

$t = 5$ $\quad = 20 \times 5 + \frac{1}{2} \times 0.8 \times 5^2$

$a = 0.8$ $\quad = 110$

$s = ?$

Total distance travelled in the interval $10 \le t \le 20$ = 210 m.

[5]

(iv) Distance travelled $= \int_0^{10} v\,dt$

$$= \int_0^{10} (4t - 0.2t^2)\,dt$$

$$= \left[2t^2 - \tfrac{1}{15}t^3\right]_0^{10}$$

$$= 200 - \tfrac{200}{3}$$

$$= 133\tfrac{1}{3}$$

Distance travelled in the interval $0 \le t \le 10$ $= 133\frac{1}{3}$ m.

[4]

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5. $v = t^2 - 4t$, $0 \le t \le 6$,

$v = c$, $6 \le t \le 10$,

$v = at + b$, $10 \le t \le 15$.

(i) When $t = 6$, $v = 6^2 - 4 \times 6 = 12$

Therefore $c = 12$.

[2]

(ii) When $t = 10$, $v = 12 \Rightarrow 10a + b = 12$

When $t = 12$, $v = 4 \Rightarrow 12a + b = 4$

Subtracting: $-2a = 8 \Rightarrow a = -4$, $b = 52$

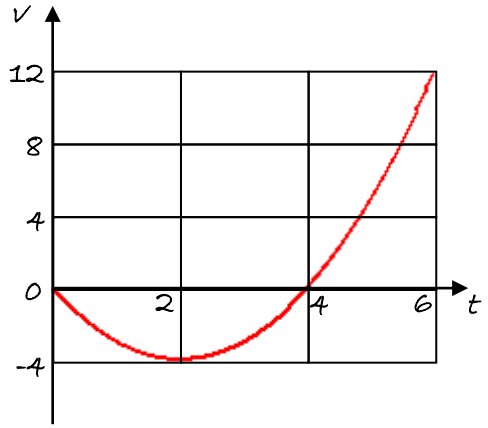
The insect is decelerating at a constant rate.

[4]

(iii) When $t = 0$, $v = 0^2 - 4 \times 0 = 0$

When $t = 2$, $v = 2^2 - 4 \times 2 = -4$

When $t = 4$, $v = 4^2 - 4 \times 4 = 0$



[3]

(iv) Displacement for $0 \le t \le 4 = \int_0^4 v\,dt = \left[\frac{1}{3}t^3 - 2t^2\right]_0^4$

$= \frac{64}{3} - 32 = -\frac{32}{3}$

Displacement for $4 \le t \le 6 = \int_4^6 v\,dt = \left[\frac{1}{3}t^3 - 2t^2\right]_4^6$

$= 72 - 72 - \frac{64}{3} + 32 = \frac{32}{3}$

Total distance travelled $= \frac{32}{3} + \frac{32}{3} = 21\frac{1}{3}$ m.

[6]